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Luisa María Gil-Martín[1] and Enrique Hernández-Montes[2]

INFLUENCE OF THE LEVER ARM IN THE STRENGTH DESIGN OF RC SLABS

Abstract

This paper presents the influence of the lever arm in the strength design of RC slabs.

The two widespread used methods in the ultimate strength design of RC slabs subjected to bending and torsional moments: the Field of Moments approach and the Sandwich method are considered in this work.

The Field of Moments method is based on Mohr´s circle representation of both capacity and demand. Once the resisting moments for each reinforcement direction are obtained, the necessary areas of steel are obtained independently.

The Sandwich method presents a general formulation in which the slab is divided in three layers. The inner one (or core) is responsible to withstand the out-plane shear while the other external actions are reduced to in-plane or membrane forces (per unit length) acting at the middle surface of the outer layers. In the Sandwich method, the areas of reinforcement in tension are obtained considering the same compression block for both directions of rebars.

In order to avoid iterations, some versions of the Sandwich method obtain the in-plane forces in the outer layers of the slab using as lever arm the distance between top and bottom reinforcement in each direction. In doing so, significant differences may exit with respect to the original method in regard to the principal compressive force (per unit of length), F_c, and to the tension forces per unit length in both reinforcement directions, N_{reinf1} and N_{reinf2}.

Differences between the Field of Moments method and two versions of the Sandwich method due to the adopted value of the lever arm are analysed in this paper.

The lever arm has been proven to be a key factor for the strength design of RC slab. Several examples are presented.

Key words

Strength design, RC slabs, lever arm influence.

[1] *Professor. Department of Structural Mechanics, University of Granada (UGR). Campus Universitario de Fuentenueva s/n. 18072 Granada, Spain, mlgil@ugr.es*

[2] *Professor. Department of Structural Mechanics, University of Granada (UGR). Campus Universitario de Fuentenueva s/n. 18072 Granada, Spain, emontes@ugr.es*

1. INTRODUCTION

The Field of Moments method (henceforth FoMM) proposed by Wood-Armer [1,2] and the Sandwich method (SM), proposed by Broundum-Nielsen [3] are two widespread methods for the ultimate strength design of RC slabs subjected to bending and torsion.

The FoMM defines the strength of a RC slab subjected to bending and torsional moments but does not consider neither shear nor axial loads. On the contrary, the SM allows for the ultimate strength design of a slab subjected to bending moments, torque, in-plane shear, out of plane shear and axial loads.

The FoMM is based on the Johansen´s normal moment yield criterion [1,2] and has a simple formulation based on continuous mechanics. In this method, the resisting moments in both directions of reinforcement necessary to guaranty that the capacity of a slab is not exceeded in flexure by loading are obtained. On the other hand, the SM considers the slab divided into three layers. In the SM, external actions (except transverse shear forces, which are withstand by the central layer [4]) are resolved into membranes forces in both reinforcement directions at the middle surfaces of the outer layers.

These methods have been included in software packages and standards: CYPE [5] and Canadian Code [6] include the FoMM [1,2] whereas that SAP2000 [7] and CEB-FIP 2010 Model Code [8] opt for the SM [3].

In this paper, it is going to be assumed that reinforcement of the slabs in both outer layer is placed in two orthogonal directions and that the slab is not subjected to transverse shear.

The lever arm is a key factor in the strength design of RC slab. In this paper its influence is studied. Several examples are presented.

2. THE FIELD OF MOMENTS MODEL (*FoMM*).

The Wood-Armer [1,2] method uses the normal moment yield criterion proposed by Johansen [9].

The applied moments expressed by the unit of length (external loading) to which the slab is subjected can be seen in Fig. 1a, in which m_{11} and m_{22} are bending moments and m_{12} is a torsional moment.

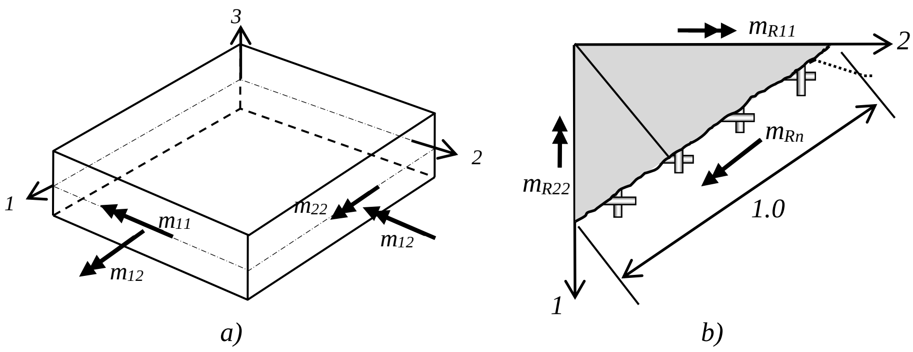

Figure 1. a) Applied actions to the shell element indicating positive sign. Adapted from [7]. *b) Resisting moment along a yield line for positive field of moments*.

The normal applied moments per unit of length along the yield line is obtained from a Mohr´s circle representation of the external field of moments (i.e. m_{11}, m_{22} and m_{12}). Regarding the uniaxial ultimate flexural moment in the reinforcement directions -per unit of length of the slab- (m_{R11} and m_{R22}, respectively), the FoMM assumes that they are principal moments and a Mohr´s circle representation is built based on them.

The normal resisting moments along a yield line whose external normal is at an angle α with axis 1 is represented in Fig.1b.

Imposing that, at any point of the slab and for all directions [10], the normal resisting moment of the slab is bigger or equal to the normal applied moment, for both positive and negative field of moments, the resisting moments in the reinforcement directions are obtained as follow:

$$\underbrace{\begin{aligned} m_{R11} &= m_{11} + |m_{12}| \\ m_{R22} &= m_{22} + |m_{12}| \end{aligned}}_{\text{Positive field of moments}} \qquad \underbrace{\begin{aligned} m_{R11} &= m_{11} - |m_{12}| \\ m_{R22} &= m_{22} - |m_{12}| \end{aligned}}_{\text{Negative field of moments}} \tag{1}$$

The uniaxial ultimate moment at yield lines per unit width, for each reinforcement direction, and for both positive and negative fields of moments can be obtained according to standard flexural strength theory [11]. In the Wood-Armer method [1,2], based on the Johansen yield criterion [9], the reinforcement crossing a yield line is supposed to yield in simple tension. So, once the design moments are known, the thickness of the compressed layer is obtained from equilibrium of axial forces, and the required reinforcement area per unit width of slab in each direction of reinforcement can be obtained by calculating the moment relative to the center of the compression block (see stress diagram in Fig.2)[11] as:

$$A_{s,i}\,f_y = f_{cd}\,c_{comp,i} \rightarrow c_{comp,i} = \frac{A_{s,i}\,f_y}{f_{cd}}$$
$$m_{Ri} = A_{s,i}\,f_y\left(d_i - 0.5c_{comp_i}\right) = A_{s,i}\,f_y\left(d_i - 0.5\frac{A_{s,i}\,f_y}{f_{cd}}\right) \rightarrow \qquad (2)$$
$$A_{s,i} = \frac{d_i\,f_{cd}}{f_y}\left(1-\sqrt{1-2\frac{m_{Ri}}{d_i^2\,f_{cd}}}\right) \quad \textit{for i=11 and 22}$$

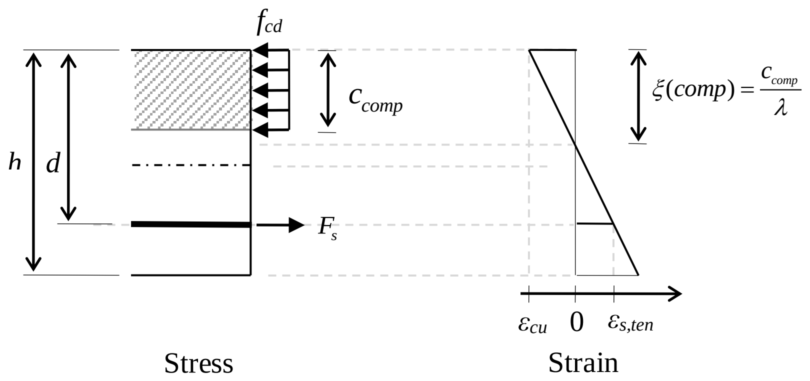

Figure 2. Strain and stress distribution in standard strength theory (positive bending).

In Eq. (2), f_{cd} is the concrete design compression strength and f_y the yield stress of the reinforcing steel.

3. THE SANDWICH MODEL (SM)

This method has been widely studied in relevant literature [8,10,11,18,27]. The *SM* considers the slab composed of three layers, see Fig. 3. The outer layers are responsible for resisting bending, torsion, axial force and in-plane shear loadings while the middle layer is responsible for resisting the out-of-plane shear (Fig.3).

The thickness of the outer layers affects the level arm and so the membrane forces. Hereafter, the mechanical covers are a fixed data.

The method proposed by Broundum-Nielsen [3] also assumes the yielding of all reinforcements crossing a crack. However, authors [14-17] proposed a verification based on deformations, similar to the strain compatibility carried out for the strength design of beams, to apply to both reinforcement directions [15] in order to obtain the actual strain in the rebars.

The original SM [3] obtains the thickness of the compressed outer layer considering the predominant bending moment while the middle of the outer layer tensioned corresponds to the centroid of the tensioned reinforcement. In doing so, the in-plane forces in each outer layer are obtained as:

$$N_{11} = \pm \frac{m_{11}}{d_{pred}};\ N_{22} = \pm \frac{m_{22}}{d_{pred}};\ N_{12} = \pm \frac{m_{12}}{d_{pred}} \tag{3}$$

with d_{pred} as the distance between the centroid of the compressed layer due to the predominant bending moment and the corresponding tensioned reinforcement.

Once the value of principal compressive force (per unit of length), F_c, is obtained from equilibrium of a portion of outer layer defined by axis 1 and 2 and a unitary length crack that forms an angle θ with 1- direction. See Fig. 4. The thickness of the compressed outer layer is obtained as F_c divided by the concrete design compression strength:

$$c_{comp} = \frac{-F_c}{f_{cd}} \quad with\ \ F_c = N_{12}\left(Tan\theta + Cot\theta\right) \tag{4}$$

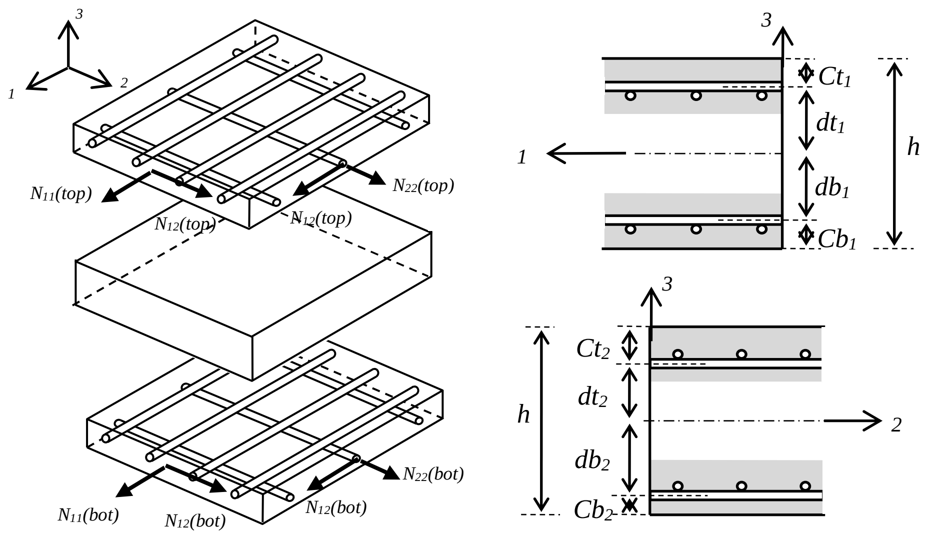

Figure 3. a) Membrane forces in a Slab Element - Sandwich model. b) Definition of variables related to the location of the reinforcement in the outer layers. Adapted from [16].

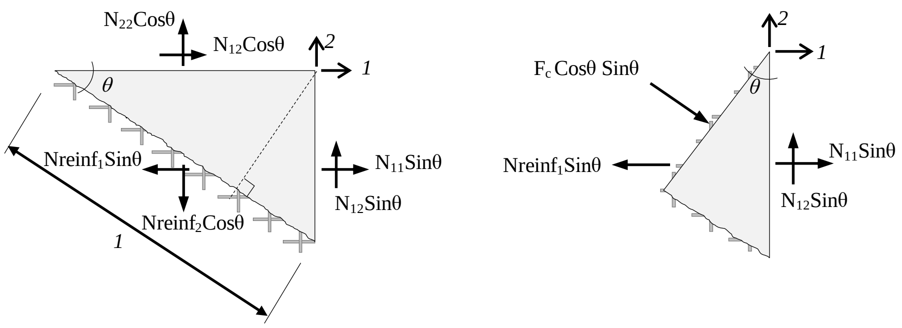

Figure 4. Equilibrium of a portion of outer layer of a slab. Adapted from [15].

When the thicknesses of the outer layers are known, the membrane forces can be calculated and the forces in the reinforcement (per unit of length) in both directions, $Nreinf_1$ and $Nreinf_2$, can be obtained from equilibrium in Fig. 4, as:

$$N\text{reinf}_1 = N_{11} - N_{12}\,Cot\,\theta \quad and \quad N\text{reinf}_2 = N_{22} - N_{12}Tan\,\theta \tag{5}$$

SAP2000 [7] introduces a modification in the original SM [3] and calculates the equivalent in-plane forces per unit of length of the slab in the covers (or outer layers) as indicated in Eq. (5) instead of from Eq.(3), see Fig. 9b.

$$N_{11} = \pm\frac{m_{11}}{db_1 + dt_1} \quad N_{22} = \pm\frac{m_{22}}{db_2 + dt_2} \quad N_{12} = \pm\frac{m_{12}}{Min[(db_1 + dt_1),(db_2 + dt_2)]} \tag{6}$$

4. COMPARISON OF SM AND FoMM IN TERMS OF THE LEVER ARM.

In Eq. (2) 3 it is clear that in the *FoMM* [1,2] the lever arms and the necessary areas of steel for each reinforcement direction are obtained independently. On the contrary, in the SM [8], the lever arm is the same for both reinforcements of each outer layer (see Eqs. (3) and (5)).

Moreover, the thickness of the compressed block due to the predominant bending in the *SM* [3] , given by Eq. (4), can be re-written as [3,14-16]:

$$c = \frac{-F_c}{f_{cd}} = -\frac{N_{12}Tan\,\theta + N_{12}Cot\,\theta}{f_{cd}} = \frac{\left(N_{\text{reinf1}} + N_{\text{reinf2}}\right) - \left(N_{11} + N_{22}\right)}{f_{cd}} \tag{7}$$

Expression (7**)** can be used for both top and bottom outer layers of the slab. Because the in-plane compressive axial force when reinforcement is required in the opposite layer (i.e.: N_{11} and/or N_{22} in Figure 3) is negative (compression), the following is true:

$$c = \frac{\left(N_{\text{reinf1}} + N_{\text{reinf2}}\right) - \left(N_{11} + N_{22}\right)}{f_{cd}} > \frac{\left(N_{\text{reinf1}} + N_{\text{reinf2}}\right)}{f_{cd}} \tag{8}$$

Regarding the *FoMM* [1,2], the thickness of the compression block can be deduced from Eq. (2), for both positive and negative bending, as:

$$c_{comp,i} = \frac{A_{s,i}\,f_y}{f_{cd}} = \frac{N_{reinf,i}}{f_{cd}} \quad \text{for i=1 and 2} \tag{9}$$

From comparison of Eqs. (8) and (9), it is clear that the depth of the compressed block corresponding to the *SM* [3] is always greater than the one obtained from the *FoMM* [1,2]. Consequently, reinforcement yielding is more probable if the Wood-Armer method [1,2] is used for the ultimate strength design of the slab. This is due to the fact that the underestimation of the thickness of the compression zone leads to the overestimation of the lever arm of the internal forces associated with the resisting moments.

The above is in agreement with remarks in [18], which state that the Wood-Armer approach [1,2] can be unsafe for slabs in bi-axial bending due to the consideration of each set of reinforcement independently.

5. INFLUENCE OF THE LEVER ARM IN THE METHODS BASED ON SANDWICH APPROACH

As commented before, SAP2000 [7] set in advance the level arm, obtaining the membranes forces from Eq. (6). Once the membrane forces are known, this commercial software applies independently the SM to each reinforcement [8,23].

In order to quantify the influence of the lever arm (*LA*) in the strength design of RC slabs, the lever arm in the slab of Table 1 has been modified as follow: both covers in the top layer (the one compressed due to the predominant bending), Ct_1 and Ct_2 in Fig. 10, have been multiplied by a parameter β. For convenience, in this case β varies from 0.05 to 2.00.

Table 1. Example 1 of slab. Adapted from [16].

External actions: $m_{11} = 100\ kNm/m$ $m_{22} = 250\ kNm/m$ $m_{12} = 50\ kNm/m$	
Geometry of the slab: $h = 300\,mm$; $Ct_1 = Cb_2 = 50\ mm$; $Ct_2 = Cb_1 = 70\ mm$	*Material:* $f_{ck} = 30\,MPa$ $(\gamma_c = 1.5)$; $f_{yk} = 500\,MPa$ $(\gamma_s = 1.15)$; λ=0.8 ϵ_{cu}=0.0035 (see Fig.2)

For the slab in Table 1, values of the principal compressive force (per unit of length), F_c, and the tension forces per unit length in both reinforcement directions, N_{reinf1} and N_{reinf2}, obtained using the *SM* [3] with the membrane forces obtained form Eq. (6) have been plotted in Figure 5 as a function of the level arm, *LA*. In Fig. 4 *LA*=Min [(h–Cb_1– βCt_1), (h–Cb_2– βCt_2)], see Fig. 3b.

a)

b)

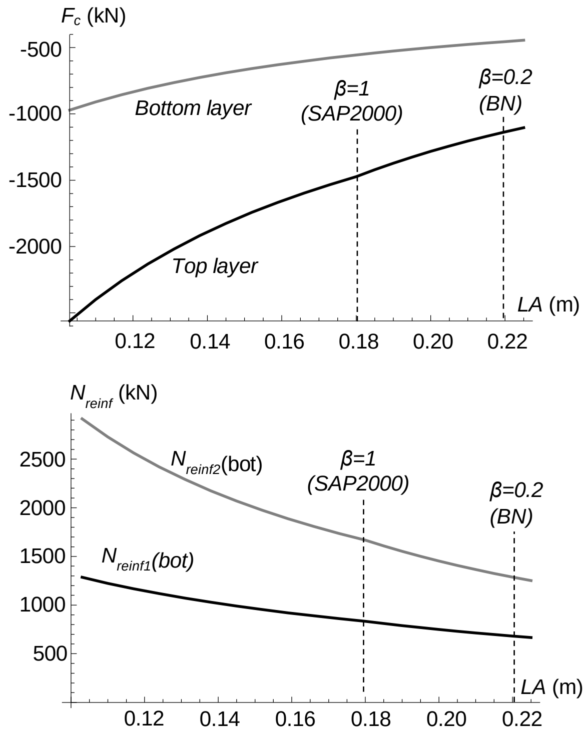

Figure 5. Influence of the level arm in the strength design of the RC slab of Table 1. a) Principal compressive force in outer layers. b) Tension forces per unit length in both reinforcement directions.

The lever arm corresponding to the procedure of SAP2000 [7] and the one obtained using the SM [3] are indicated in Figure 5. This figure shows that the lever arm has a great influence in the strength design of RC slabs.

As can be seen in Fig. 5, the application of the SM independent for each direction using the membrane forces given by Eq. (5), as SAP2000 [7] does, leads to a significant increase of the necessary area of reinforcement respect to the original method proposed by Broundum-Nielsen [3] (around 60% in the case of slab in Table 4, see Fig. 5) [16].

6. CONCLUSIONS

A clear difference exits between the way in which the depth of the compression block in the slab is obtained. In the FoMM [6,7] a different depth is associated with each set of reinforcement, leading to a lower thickness of the compression block than in the case of the SM [8], in which an unique depth for both sets of reinforcement is considered. This is relevant because both the lever arm of the internal forces and the strain at the reinforcement can be unsafely affected.

In order to avoid iterations, some attempts have been done of applying the Broundum-Nielsen formulation [8] individually as in the software SAP2000 [7]. To do so the lever arm needs to be set in advance. This approach must be applied with caution because it has been proved in this paper the lever arm has a great influence of the strength design of RC slab.

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