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DESIGN OF A COMPRESSION STRUCTURE WITH PRESTRESSING TENDONS INSPIRED BY THE TWA FLIGHT CENTER

Abstract

Tension- and compression-only structures can be modelled as pin-jointed networks with all its member in tension or in compression respectively. The equilibrium configuration can be obtained using well-known form-findings methods such as the Force Density Method (FDM).

Due to the materials employed in the construction of a tension structure (in general woven fabric), their self-weight is ignored during the design. By doing so, the corresponding force density or force:length ratio matrix is non-singular and the final shape is easily obtained from the equilibrium equations.

Regarding compression-only structures such as vaults and domes, they can also be modelled as pin-jointed networks. Now the self-weight is the dominant load and it cannot be ignored, which leads to the loss of linearity of the problem because it depends on the final shape of the structure. However, there are some iterative methods to overcome this issue.

Compression structures with prestressing tendons are a new type of structures introduced by the authors in a previous work. Prestressing tendons are introduced in compression-only structures, allowing the designer to obtain more creative equilibrium configurations. The prestressing tendons are modelled as tension members. Due to the coexistence of compression and tension members the corresponding force density matrix can be singular or ill-conditioned and the solution is not quite simple as in the case of tension- and compression-only structures.

In this work a compression structure with prestressing tendons is studied. The influence of the topological sequence used for the definition of the mesh of the structure is studied. The model obtained from the form-finding method is introduced in a finite element analysis software (SAP2000®).

Key words

Compression structures; Form-finding; Prestressing tendons; Force Density Method

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1. INTRODUCTION

Vaults and domes can be considered as compression structures because they carry their loads mainly in compression. This type of structures can be modelled as pin-jointed networks whose members are in compression [1-3]. On the other hand, tension-only structures (for example fabric structures) can also be modelled as pin-jointed networks but in this case with all the members in tension. The Force Density Method (FDM) introduced by Linkwitz and Schek [4-5] is a well-known procedure used to solve the form-finding problem of pin-jointed networks. The key concept of FDM is the force:length ratio or force density q, which is defined as the ratio between the axial force and the length of each member of the network. The non-linear equilibrium equations of the pin-jointed network are linearized introducing given values of q for all the elements.

The first step in the form-finding process is the definition of the mapping of the network. Topological Mapping (TM) proposed by Hernández-Montes et al. [6] is a mapping method for the definition of the connectivity between the nodes of a general mesh. A mesh of triangles is generated based on some topological rules defined in [6]. The main advantage of the method is that the mesh is defined regardless of the final shape of the structure.

In general, the self-weight of tension structures is neglected and the final shape is easily obtained from the equilibrium equations. In the case of compression structures, the introduction of the self-weight as external forces applied at the nodes of the mesh in FDM leads to a loss of the linearity of the form-finding problem. This problem is solved by an iterative procedure proposed by Carbonell-Márquez et al. [1] for compression structures. Fernández-Ruiz et al. [2] extended the iterative procedure for compression structures with inner ribs. Ribs are groups of branches of a mesh with a higher value of force:length ratio (which means that they carry a higher load level).

In tension structures all the elements are in tension, which correspond with $q > 0$. By the contrary, in compression structures all the elements or branches have $q < 0$ (compression). For this reason, compression structures can be computed as the inverse of a tension structure by geometrical inversion with respect to the horizontal plane. This procedure is very similar to the physical models used by the Spanish architect Antonio Gaudí [7].

Compression structures with prestressing tendons are a new type of structures recently introduced by the authors [8]. They are formed by a compression-only structure and tension members connected between two different points of the mesh or a point of the mesh and an external fixed point. The introduction of punctual tension members offers endless design possibilities of compression structures. However, this new type of structures cannot be considered as a tension structure neither a compression structure because compression ($q < 0$) and tension ($q > 0$) members coexist. This is the main reason why compression structures with prestressing tendons cannot be designed using existing form-finding methods. In Fernández-Ruiz et al. [8] a form-finding method for compression structures with prestressing tendons based on TM-FDM was proposed. Some requirements of the rank and conditioning of the force density matrix are imposed in order to obtain an acceptable structure.

In this work, the influence of the topological sequence used in TM for the definition of the mesh in the conditioning of the force density matrix is studied. A structure inspired by the TWA Flight Center at New York City's John F. Kennedy International Airport is presented. Finally, the model obtained from the form-finding method is introduced in a finite element analysis software

(SAP2000®) in order to analyse the presence of bending moments in the model and the level of tensile stress in the shell.

2. EQUILIBRIUM EQUATIONS IN FDM AND REQUIREMENTS OF THE FORCE DENSITY MATRIX D

In this work, the same terminology found in [5] is used. Two types of nodes are identified: free (which are free to move in the space) and fixed (which acts as supports). The total number of nodes n_s is the sum of free (n) and fixed (n_f) nodes. The connectivity matrix $\mathbf{C_S}$ ($m \times n_s$) of a general mesh with m members define the connectivity between the nodes of the mesh. If a branch j links the nodes $i(j)$ and $k(j)$ (with $i < k$), $\mathbf{C_S}$ is defined as follows:

$$\mathrm{C}_s = \begin{cases} +1 & \text{if } r = i(j) \\ -1 & \text{if } r = k(j) \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

In Eq. (1) r denotes the rth column of the jth row in $\mathbf{C_S}$. The connectivity matrix $\mathbf{C_S}$ is defined using the topological sequences shown in TM [6]. There are three basic types of topological relationships: A, B and C (see [6] for further details).

In general, fixed nodes are taken at the end of the sequence. By doing so, $\mathbf{C_S}$ can be partitioned into two matrices as $\mathbf{C_S} = [\mathbf{C}, \mathbf{C_f}]$, where $\mathbf{C}$ $[m \times n]$ and $\mathbf{C_f}$ $[m \times n_f]$ describe the connectivity of free and fixed nodes, respectively. Now the following vectors are defined:

$\mathbf{x}$, $\mathbf{y}$ and $\mathbf{z}$: coordinates of free nodes

$\mathbf{x_f}$, $\mathbf{y_f}$ and $\mathbf{z_f}$: coordinates of fixed nodes

$\mathbf{P_x}$, $\mathbf{P_y}$ and $\mathbf{P_z}$: external loads applied at the free nodes

$\mathbf{l}$,$\mathbf{s}$ and $\mathbf{q}$: length, internal force and force length ratio (l/s) of each branch of the mesh

The equilibrium equations of a general network are the following [5]:

$$\left.\begin{aligned} \mathbf{C}^{\mathrm{T}}\mathbf{QCx} + \mathbf{C}^{\mathrm{T}}\mathbf{QC}_{\mathrm{f}}\mathbf{x}_{\mathrm{f}} = \mathbf{P}_{\mathrm{x}} \\ \mathbf{C}^{\mathrm{T}}\mathbf{QCy} + \mathbf{C}^{\mathrm{T}}\mathbf{QC}_{\mathrm{f}}\mathbf{y}_{\mathrm{f}} = \mathbf{P}_{\mathrm{y}} \\ \mathbf{C}^{\mathrm{T}}\mathbf{QCz} + \mathbf{C}^{\mathrm{T}}\mathbf{QC}_{\mathrm{f}}\mathbf{z}_{\mathrm{f}} = \mathbf{P}_{\mathrm{z}} \end{aligned}\right\} \tag{2}$$

where $\mathbf{Q}$ is the diagonal matrix of the vector $\mathbf{q}$. The force density or force:length ratio matrices are defined as $\mathbf{D} = \mathbf{C}^{\mathrm{T}}\mathbf{QC}$ $[n \times n]$ and $\mathbf{D_F} = \mathbf{C}^{\mathrm{T}}\mathbf{QC_f}$ $[n \times n_f]$. Taking these definitions into account, Eq. (2) can be rewritten as:

$$\left.\begin{aligned} \mathbf{D}\,\mathbf{x} &= \mathbf{P}_x - \mathbf{D}_f\,\mathbf{x}_f \\ \mathbf{D}\,\mathbf{y} &= \mathbf{P}_y - \mathbf{D}_f\,\mathbf{y}_f \\ \mathbf{D}\,\mathbf{z} &= \mathbf{P}_z - \mathbf{D}_f\,\mathbf{z}_f \end{aligned}\right\} \tag{3}$$

As in compression structures with prestressing tendons compression and tension members coexist, matrix **D** can be singular or ill-conditioned. For this reason and according to the prescriptions given in [8], matrix **D** should be (1) non-singular and (2) well-conditioned (in that order).

According to [8], **D** is well-conditioned if its condition number k is lower than k_{lim}. In [8], k_{lim} was defined as the condition number for which a relative change of 1% in the force:length ratios of the prestressing tendons leads to a relative change in any of the coordinates x, y or z that is less than 1%, and very close to 1%. This values is unique for each structure and it depends on the force:length ratio distribution and the presence of ribs and prestressing tendons, among others.

3. COMPRESSION STRUCTURE WITH PRESTRESSING TENDONS INSPIRED BY TWA TERMINAL

In this work, a compression structure with prestressing tendons inspired by the TWA Flight Center at New York City's John F. Kennedy International Airport (see Figure 1) is presented. The terminal was opened in 1962 and it was designed by the Architect Eero Saarinen. The structure has a distinctly birdlike design in order to symbolize air travelling [9]. The number and coordinates of the supports of the compression structure presented in this paper are the same than those of the TWA terminal.

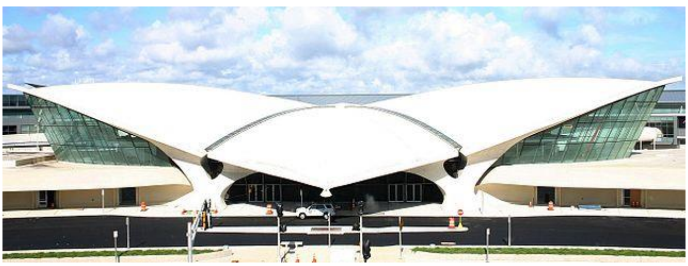


Figure 1. TWA Terminal [10]

3.1. INFLUENCE OF THE TOPOLOGICAL SEQUENCE IN K_{LIM}

Two compression structures with prestressing tendons inspired by the TWA Terminal (see Figure 1) are designed. Both structures have approximately the same ribs distribution and the same q for all the members (for both compressed and tensioned members). The only difference is that the topological sequence (according to TM [6]) of TWA-1 structure is ACCACCACCAC and the one of TWA-2 is ACACACCCCCC. In both cases the self-weight of the structure is γ = 4.71 kN/m^2

(corresponding to a concrete slab of 20 cm thick) and the self-weight of the compression ribs is λ = 1.41 kN/m (corresponding to concrete ribs with 20 × 30 cm section approx.).

The compression structure is formed by 12 rings, 4 fixed nodes and 12 cables. Figures 2.a and 2.b show the assignation of the force:length ratio coefficients to all the members of the mesh of TWA-1 and TWA-2. Due to the different topological sequence, the number of nodes and elements of the mesh of TWA-1 is different than the ones of TWA-2. The force density matrices of both structures are computed as $\mathbf{D} = \mathbf{C}^T\mathbf{Q}\mathbf{C}$.

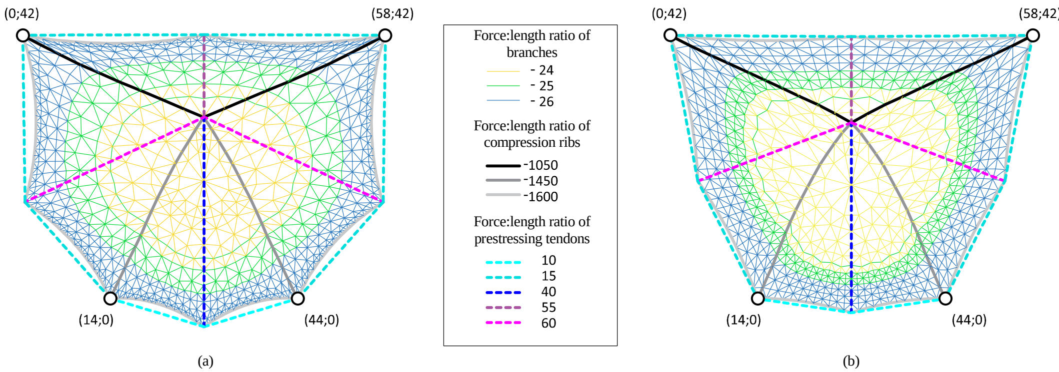
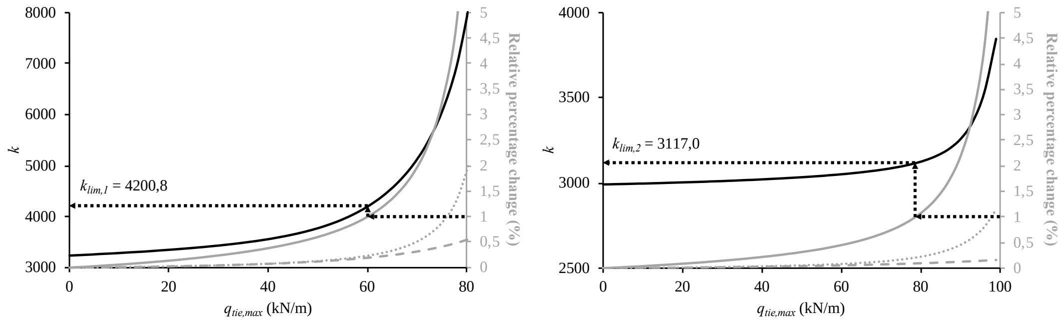


Figure 2. TWA-1 (a) and TWA-2 (b) meshes and assignation of the force:length ratio values to all the elements

Once defined, the force density matrices **D** of both structures are checked according to the two conditions defined in [8]. The first condition is that matrix **D** must be non-singular: this condition is fulfilled in both examples. The second condition is that the condition number of **D** (k) must be lower than k_{lim}. The condition number k depends on the force density matrix **D** and it is given by (see [8]):

$$k = \frac{\sqrt{\lambda_{\max}}}{\sqrt{\lambda_{\min}}} \tag{4}$$

It can be proved that the values of k corresponding to the two studied examples (TWA-1 and TWA-2) are 4193.6 and 3051.2 respectively. Figure 3 shows the variation of the value of k with the increment of the force:length ratio of the ties. For simplicity, only the maximum value of the q of the ties ($q_{tie,max}$) is considered in the graph, the rest of the q values of the ties are increased proportionally (see Figure 3). In this paper k_{lim} is calculated according to the prescriptions given in [8] considering, as commented before, as acceptable a change in the geometry of the structure less than 1% when a relative change of 1% in the force:length ratios of the prestressing tendons occurs. For the structures studied $k_{lim,1}$ = 4200.8 and $k_{lim,2}$ = 3117.0, for TWA-1 and TWA-2 respectively.



(a) (b)

Figure 3. Relative percentage change of the modules of vectors **x**, **y** *and* **z** *due to a relative change of 1% in* $q_{tie,max}$ *for the TWA-1 (a) and TWA-2 (b)*

Comparison of k and k_{lim} shows that in both cases $k < k_{lim}$ and so both structures are well-conditioned. However, $k_{lim,1}$ is higher than $k_{lim,2}$, so the topological sequence of the mesh has a noticeable influence on the value of k_{lim} (and obviously on k). In addition, the maximum height of the resultant structure is also affected by the topological sequence: $z_{max,1}$ = 10.11 m (TWA-1) and $z_{max,2}$ = 8.67 m (TWA-2).

Henceforth a more detailed studied of the highest structure, that is TWA-1, is presented. Figure 4 shows a view of the equilibrium configuration of TWA-1 example.

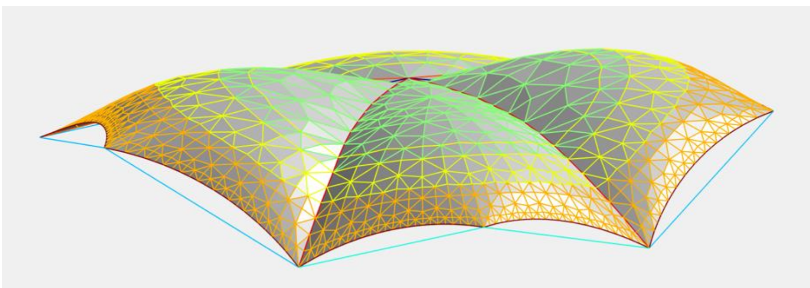
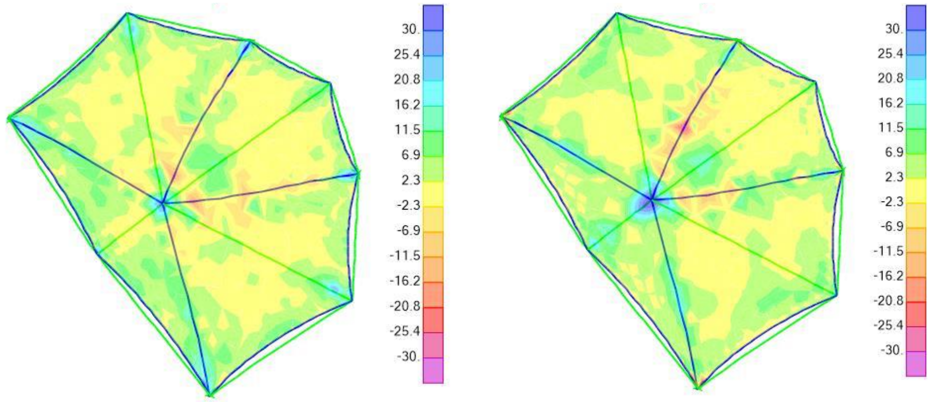


Figure 4. TWA-1 equilibrium configuration

3.2. BENDING MOMENTS IN COMPRESSION STRUCTURES WITH PRESTRESSING TENDONS

The mesh of the TWA-1 example, containing both cables and ribs, is now imported to be analysed with the finite element analysis software SAP2000. The structure is modelled with homogeneous thin-shell elements of 20 cm thickness, which neglects transverse shear deformation and can withstand both, in plane and out of plane forces. Ribs have concrete rectangular cross-sections 20 cm width and 30 cm height. Material of both structure and ribs is assumed to be concrete with f_{ck} = 40 MPa, with a tensile strength equal to f_{ctm} = 3,51 MPa (computed according Eurocode 2 [11] prescriptions). External cables are modelled with steel cable elements, 10 mm diameter. Tension, obtained as $q \cdot l$, is applied at one of the ends of each cable.

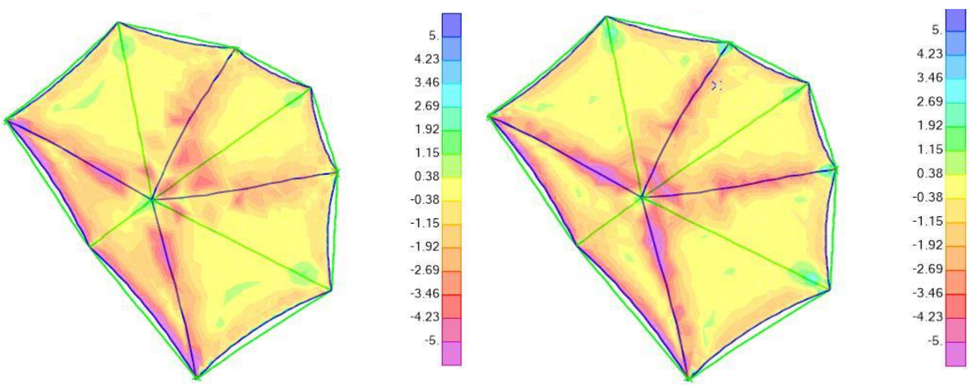
Values of the principal moments in shell elements corresponding to the mid-plane of the shell are presented in Figure 5. As can be seen in Figure 5 moments are very small in the structure, except in the central point of confluence of ribs and cables and in some locations near the ribs, where values close to 30 kN·m are reached (in blue tones in Figure 5).



a) b)

Figure 5. Principal moments [kN·m] in the mid-plane of shell elements. a) maximum and b) minimum

Figure 6 corresponds to the maximum –tensile– principal stress at both top and bottom faces of the shell. As can be seen in Figure 6, most of the structure at both sides is in compression. Tension is located next to the ends of the cables (green-blue colour in sketches in Figure 6).



a) b)

Figure 6. Maximum principal stress [MPa] in the shell elements. a) top face and b) bottom face. Positive means tension

The maximum value of the tension stress is 3.5 MPa, and it is located next to the ends of the cables. So, cracking of concrete in the shell occurs in these zones, where special measures must be taken in order to tie the cables.

4. CONCLUSIONS

Compression structures with prestressing tendons are a new type of structures introduced by the authors [9]. They can be modelled as a pin-jointed network (compression structure) with punctual tension members between two nodes of the mesh (prestressing tendons). The introduction of prestressing tendons gives to the designer more creative opportunities in the design of compression structures. The following conclusions can be drawn for the present work:

1- Emblematic buildings can serve as an inspiration of compression structures with prestressing tendons. In this work, the TWA Terminal has been taken as a model for the design of the structure. A geometry similar to that of the TWA Terminal cannot be obtained without the introduction of prestressing tendons.

2- Based on the examples developed in this work it can be concluded that the topological sequence adopted in TM [6] has a noticeable influence on the value of k_{lim}. Moreover, a significant influence in both the value of k and the height of the structure was also noticed. More research is needed in order to both evaluate and quantify this effect.

3- The load pattern including self-weight of the structure and the tension in cables provokes tension stresses in the range of the average tension strength of concrete in the zones of the shell closer to the ends of the cables. However, the finite element analysis also shows that most of the structure at both top and bottom sides is in compression.

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