Soluciones autosemejantes del flujo lagrangiano de la curvatura media

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Guión

1. Flujo de la curvatura media
   - Definición del FCM
   - Soluciones autosemejantes del FCM
   - Resultados en codimensión 1
2. Contexto lagrangiano
   - Superficies lagrangianas en el plano euclídeo complejo
   - Superficies lagrangianas estacionarias hamiltonianas (LEH)
   - Flujo lagrangiano de la curvatura media
3. Ejemplos de soluciones autosemejantes del FCM lagrangiano
4. Soluciones autosemejantes estacionarias hamiltonianas
   - Existencia
   - Unicidad

Primera variación del volumen

$\phi : M^n \to \mathbb{R}^m$ inmersión

H vector curvatura media, $\Delta\phi = n\,H$

$F : (-\epsilon, \epsilon) \times M \to \mathbb{R}^m$, $F_t = F(t, .)$, variación de $\phi = F_0$

$A(t) = \int_M dv_t$, $A'(0) = -n \int_M H \cdot V\, dv_0$, $V = F_*(\partial_t)|_{t=0}$

Puntos críticos del volumen para variaciones *arbitrarias*:
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Φ mínima $\Leftrightarrow H = 0$

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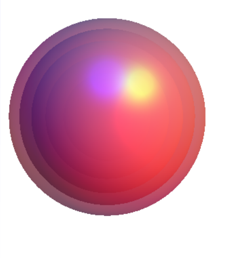

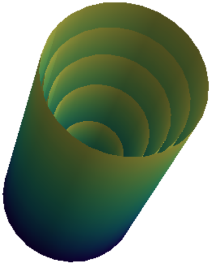

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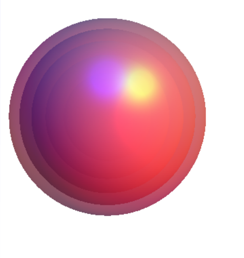

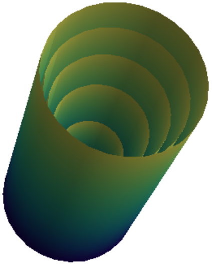

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$F_t(p) = \sqrt{2at+1}\,\phi(p), \; 2at+1 > 0,$ solución (FCM) si $H = a\,\phi^\perp$

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Superficies lagrangianas en $\mathbb{C}^2$

$(\mathbb{R}^4 \equiv \mathbb{C}^2, \langle , \rangle, J)$ plano euclídeo complejo, $\omega(.,.) = \langle J.,. \rangle = \frac{1}{2} d\lambda$

$\Phi : M^2 \rightarrow \mathbb{C}^2$ lagrangiana si $\Phi^* \omega = 0 \Leftrightarrow J : TM \cong T^\perp M$

$\sigma(v,w) = JA_{Jv}w$, $C(\cdot,\cdot,\cdot) = \langle \sigma(\cdot,\cdot), J\cdot \rangle$ simétrica

1-forma curvatura media: $\alpha_H = H \lrcorner \omega = \langle JH, . \rangle \quad d\alpha_H = 0$

$\phi^*(dz_1 \wedge dz_2) = e^{i\beta}\omega_M$, $\beta : M \rightarrow \mathbb{R}/2\pi\mathbb{Z}$ ángulo lagrangiano;

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✓[Minicozzi, 1995] Conjetura de Willmore restringida a los toros lagrangianos de $\mathbb{C}^2 \Rightarrow$ Conjetura de Oh

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$\phi : M^2 \to \mathbb{R}^4 \equiv \mathbb{C}^2$ lagrangiana, F_t FCM de ϕ

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ϕ lagrangiana $\Rightarrow F_t$ lagrangiana, $\forall t \in [0, T_{sing})$, $0 < T_{sing} < \epsilon$

Singularidades en $T_{sing} \to$ soluciones tipo soliton

☒ *Soluciones autosemejantes* del FCM lagrangiano:
$(H = a\,\phi^\perp \Leftrightarrow \alpha_H = 2a\,\phi^*\lambda)$
Ejemplos más sencillos:

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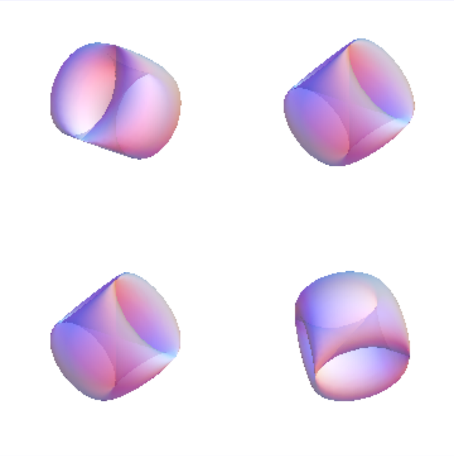
Proyecciones en 3-espacios coordenados del toro de Clifford



Producto de curvas planas

Ejemplos 1: $\alpha_1 \times \alpha_2$

$\alpha_1 = \alpha_1(t) \in \mathbb{C}$, $\alpha_2 = \alpha_2(s) \in \mathbb{C}$, ambas p.p.a.

$$\phi(t,s) = (\alpha_1(t), \alpha_2(s))$$

$$2H = \kappa_{\alpha_1} J\phi_t + \kappa_{\alpha_2} J\phi_s$$

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Soluciones autosemejantes del FCM lagrangiano en $\alpha_1 \times \alpha_2$

$\phi = \alpha_1 \times \alpha_2$ autosemejante (i.e. $H = a\,\phi^\perp$)

$$\Leftrightarrow \kappa_{\alpha_i} = 2a\,\alpha_i^\perp,\ i = 1,2 \Leftrightarrow \kappa_{\alpha_i} = -2a\sqrt{E_i}\,e^{-a|\alpha_i|^2},\ i = 1,2$$

$$0 \leq E_i = r_i^2(1 - r_i'^2)\,e^{2ar_i^2},\ r_i = |\alpha_i|,\ i = 1,2$$

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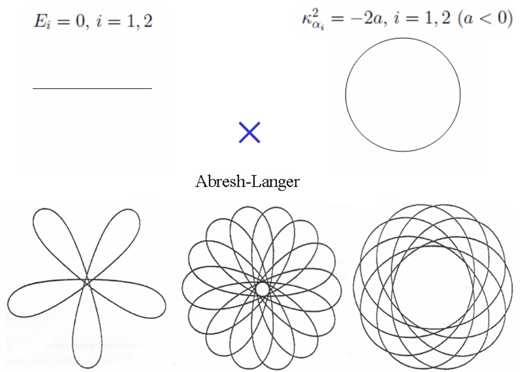

Producto de curva plana y curva de Legendre esférica

Ejemplos 2: $\alpha \cdot \xi$ ✓[Ros & Urbano, 1998]

$\alpha = \alpha(t) \in \mathbb{C}^*$, $\xi = \xi(s) \subset \mathbb{S}^2\left(\frac{1}{2}\right)$ p.p.a.

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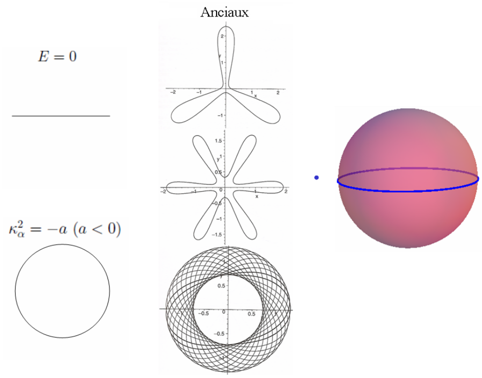

“Producto” de curvas de Legendre esférica e hiperbólica

Ejemplos 3: $\eta \odot \xi$ ✓[- & Chen, 2008]

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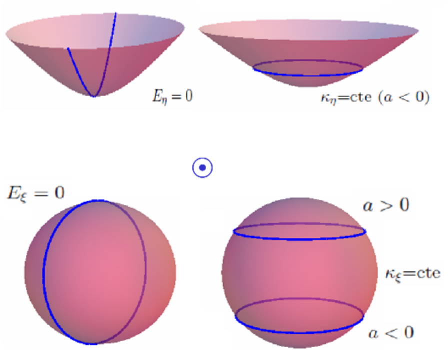

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$$\Phi_\delta(s,t) = \frac{1}{\sqrt{2a}} \left(i\, \mathrm{s}_\delta \cosh t\, e^{-\frac{is}{\mathrm{c}_\delta}}, \mathrm{t}_\delta \sinh t\, e^{i\, \mathrm{c}_\delta\, s} \right)$$

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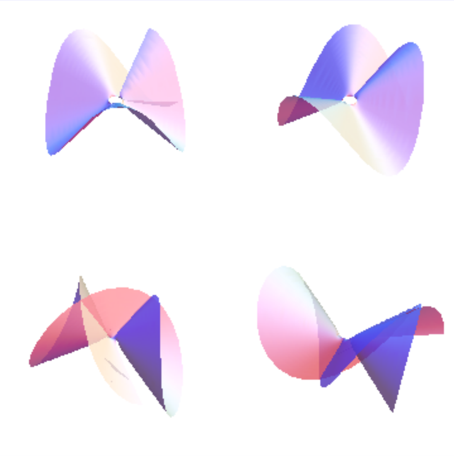
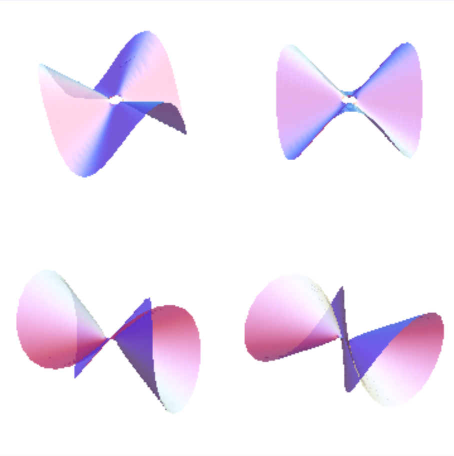
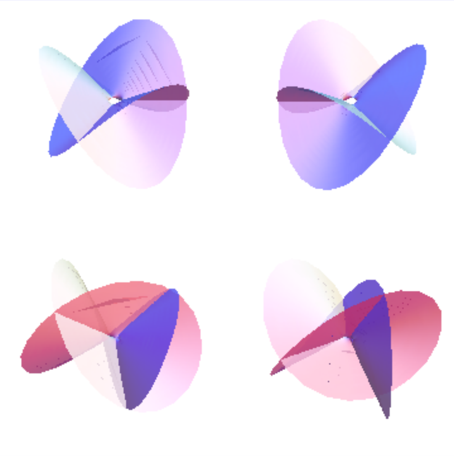
Plano Φ_δ, $\cosh^2 \delta \notin \mathbb{Q}$



Cilindro $\Phi_{3,1}$



Cinta de Moebius $\Phi_{3,2}$



Familia $\Upsilon_\gamma : \mathbb{R}^2 \to \mathbb{C}^2$, $0 < \gamma < \pi/2$

$$\Upsilon_\gamma(s,t) = \frac{1}{\sqrt{-2a}} \left(-i\, \mathrm{s}_\gamma \cosh t\, e^{\frac{i s}{\mathrm{c}_\gamma}}, \mathrm{t}_\gamma \sinh t\, e^{-i\, \mathrm{c}_\gamma s} \right)$$

$\mathrm{s}_\gamma = \sin\gamma$, $\mathrm{c}_\gamma = \cos\gamma$, $\mathrm{t}_\gamma = \tan\gamma$ LEH $H_\gamma = a\,\Upsilon_\gamma^\perp$, $a < 0$

$\cos^2\gamma \notin \mathbb{Q}$, Υ_γ plano embebido; $\cos^2\gamma = p/q \in \mathbb{Q}$, $(p,q) = 1$

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- q par, p impar: $\Upsilon_{p,q}(s + \pi\sqrt{pq}, -t) = \Upsilon_{p,q}(s,t)$, $\forall (s,t) \in \mathbb{R}^2$ cinta de Moebius

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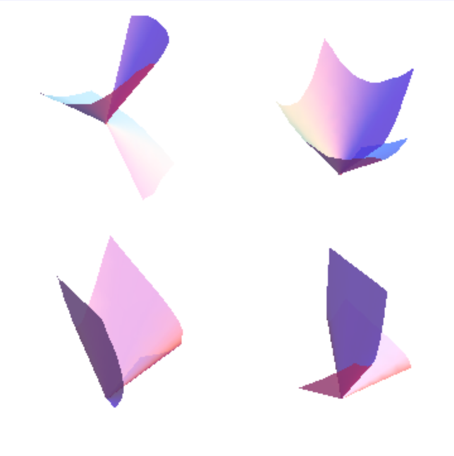
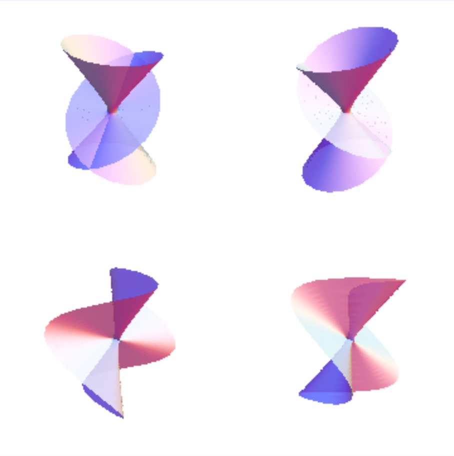
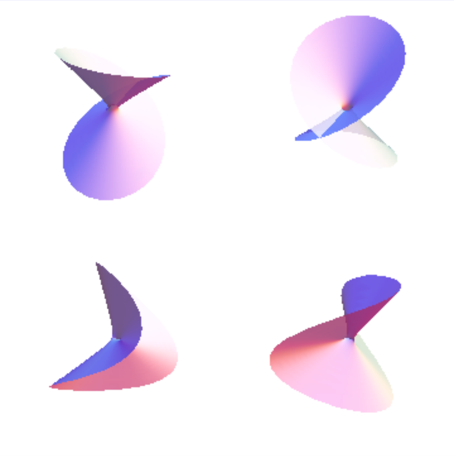
Plano Υ_γ, $\cos^2\gamma \notin \mathbb{Q}$



Cilindro $\Upsilon_{1,3}$



Cinta de Moebius $\Upsilon_{1,2}$



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Familia $\Psi_{m,n}$ (cont.)

- m impar, n par: $\Psi(2\pi - s, t + \pi\sqrt{mn}) = \Psi(s,t)$, $\forall (s,t) \in \mathbb{R}^2$
  botella de Klein
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Toro de Clifford $\mathcal{T}_{1,1}$ único embebido

$$\mathrm{Area}(\mathcal{T}_{m,n}) = \begin{cases} \frac{(m+n)^2\pi^2}{-a\sqrt{mn}}, & m \text{ o } n \text{ par} \\ \frac{(m+n)^2\pi^2}{-2a\sqrt{mn}}, & m \text{ y } n \text{ impar} \end{cases}$$

$$\mathrm{Willmore}(\mathcal{T}_{m,n}) = \begin{cases} \frac{(m+n)^2\pi^2}{\sqrt{mn}}, & m \text{ o } n \text{ par} \\ \frac{(m+n)^2\pi^2}{2\sqrt{mn}}, & m \text{ y } n \text{ impar} \end{cases}$$

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Familia $\Psi_{m,n}$ (cont.)

- m impar, n par: $\Psi(2\pi - s, t + \pi\sqrt{mn}) = \Psi(s,t)$, $\forall (s,t) \in \mathbb{R}^2$
  botella de Klein
- m par, n impar: $\Psi(\pi - s, t + \pi\sqrt{mn}) = \Psi(s,t)$, $\forall (s,t) \in \mathbb{R}^2$
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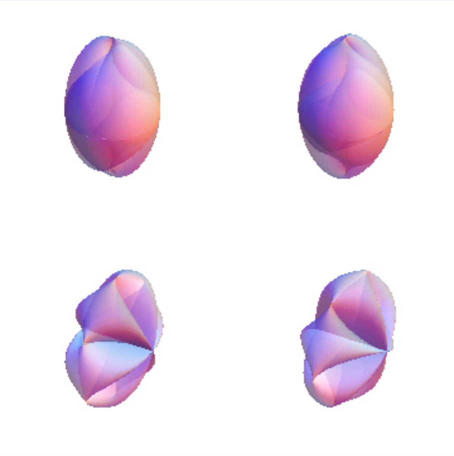
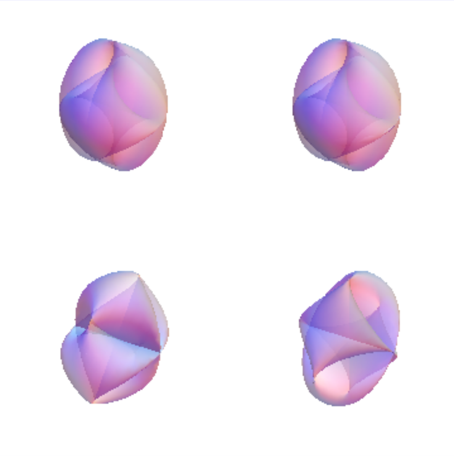
Toro $\Psi_{1,3}$



Botella de Klein $\Psi_{1,2}$



Clasificación

Teorema

$\phi : M^2 \to \mathbb{C}^2$ *LEH solución autosemejante de (FCM)*

(a) ϕ *expansiva* $(H = a\phi^\perp,\ a > 0)$

$\Rightarrow \phi \overset{loc}{\sim} \Phi_\delta : \mathbb{R}^2 \to \mathbb{C}^2,\ \delta > 0$

(b) ϕ *contráctil* $(H = a\phi^\perp,\ a < 0)$

$\Rightarrow \phi \overset{loc}{\sim}$

(i) $\mathbb{S}^1(\frac{1}{\sqrt{-2a}}) \times \mathbb{R}$

(ii) $\mathbb{S}^1(\frac{1}{\sqrt{-2a}}) \times \mathbb{S}^1(\frac{1}{\sqrt{-2a}})$

(iii) $\Upsilon_\gamma : \mathbb{R}^2 \to \mathbb{C}^2,\ 0 < \gamma < \pi/2$

(iv) $\Psi_\nu : \mathbb{S}^1 \times \mathbb{R} \to \mathbb{C}^2,\ \nu > 0$

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Esbozo de la demostración I

$z = x + iy$, $\langle , \rangle = e^{2u(z)}|dz|^2$

$$\Theta(z) = f(z)(dz)^3, \;\; f(z) = 4C(\partial_z, \partial_z, \partial_z)$$

$$\Lambda(z) = \bar{h}(z)dz, \;\; h(z) = 2\omega(\partial_{\bar{z}}, H)$$

$$\phi_{zz} = 2u_z\phi_z + \frac{\bar{h}}{2}J\phi_z + \frac{e^{-2u}f}{2}J\phi_{\bar{z}}$$

$$\phi_{z\bar{z}} = \frac{h}{2}J\phi_z + \frac{\bar{h}}{2}J\phi_{\bar{z}},$$

$$4u_{z\bar{z}} + \frac{|h|^2 - e^{-4u}|f|^2}{2} = 0$$

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Esbozo de la demostración III

E.D.O. de $g = g(y) := |\phi|^2 > 0$

$$a^2(gg'' - g'^2) = 4\mu^2(1 + ag)$$

♦ Solución constante: $g \equiv -1/a$, $a < 0$; $e^{2u} = -\mu^2/a$, $f = \mu^3/a$

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Esbozo de la demostración III

E.D.O. de $g = g(y) := |\phi|^2 > 0$

$$a^2(gg'' - g'^2) = 4\mu^2(1 + ag)$$

♦ Solución constante: $g \equiv -1/a$, $a < 0$; $e^{2u} \equiv -\mu^2/a$, $f \equiv \mu^3/a$
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Esbozo de la demostración IV

$$\phi_{xx} = -u'\phi_y + \left(2\mu - \frac{\mu E}{a} e^{-2u}\right) J\phi_x$$

$$\phi_{xy} = u'\phi_x + \frac{\mu E}{a} e^{-2u} J\phi_y$$

$$\phi_{yy} = u'\phi_y + \frac{\mu E}{a} e^{-2u} J\phi_x$$

... $\phi_{xyy} = E\phi_x$, $\phi_{yyy} = E\phi_y$. Salvo traslaciones:

$$\phi_{yy} = E\phi, \quad \phi_{xx} = -E\phi + 2\mu J\phi_x$$

Esbozo de la demostración IV

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Esbozo de la demostración V

- $E = 0 \Rightarrow a < 0$: $e^{2u} = -2\mu^2/a$, $f = -2\mu^3/a$;

  $\phi_{xx} = 2\mu J\phi_x$, $\phi_{xy} = \phi_{yy} = 0 \rightsquigarrow \mathbb{S}^1(\frac{1}{\sqrt{-2a}}) \times \mathbb{R}$

- $E > 0$

$\phi(x,y) = \cosh(\sqrt{E}y)C_1(x) + \sinh(\sqrt{E}y)C_2(x)$

$= \left(\frac{i\mu}{a\sqrt{\alpha}}\cosh(\sqrt{E}y)\exp\left(-\frac{ia\alpha}{\mu}x\right), \frac{\sqrt{\alpha}}{\sqrt{E}}\sinh(\sqrt{E}y)\exp\left(\frac{i\mu E}{a\alpha}x\right)\right)$

$b := g(0) > 0$, $x + iy = \frac{1}{\sqrt{E}}(s + it)$

$\phi_b(s,t) =$
$\left(\pm i\sqrt{b}\cosh t \exp\left(\frac{\mp i s}{\sqrt{1+2ab}}\right), \frac{\sqrt{b}}{\sqrt{1+2ab}}\sinh t \exp\left(\pm i\sqrt{1+2ab}\, s\right)\right)$
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Esbozo de la demostración V

- $E = 0 \Rightarrow a < 0$: $e^{2u} \equiv -2\mu^2/a$, $f \equiv -2\mu^3/a$;

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- $a<0$: $b=\frac{\sin^2\gamma}{-2a}$, $0<\gamma<\pi/2 \rightsquigarrow \Upsilon_\gamma$

Esbozo de la demostración VI

♦ $-\mu^2 \leq E < 0$

$\phi(x,y) = \cos(\sqrt{-E}y)C_1(x) + \sin(\sqrt{-E}y)C_2(x)$

$= \left(\frac{i\mu}{a\sqrt{\alpha}} \cos(\sqrt{-E}y) \exp\left(\frac{-ia\alpha}{\mu}x\right), \frac{\sqrt{\alpha}}{\sqrt{-E}} \sin(\sqrt{-E}y) \exp\left(\frac{i\mu E}{a\alpha}x\right)\right)$

$$c := g(0) > 0,\ c > -1/2a;\ x + iy = \frac{1}{\sqrt{-E}}(t + is)$$

$\phi_c(t,s) =$
$\left(-i\sqrt{c}\cos s \exp\left(\frac{i\,t}{\sqrt{-1-2ac}}\right), \frac{\sqrt{c}}{\sqrt{-1-2ac}} \sin s \exp\left(i\sqrt{-1-2ac}\,t\right)\right)$

$$c = \frac{\cosh^2 \nu}{-2a},\ \nu > 0 \rightsquigarrow \Psi_\nu$$

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MUCHAS GRACIAS

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