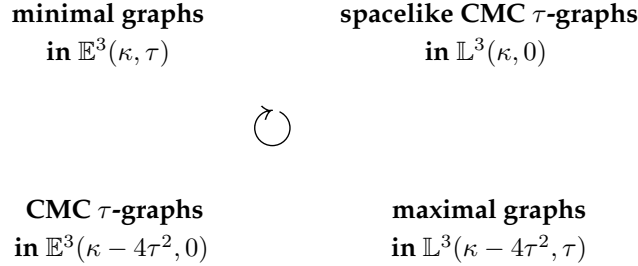


## TWO GENERALIZATIONS OF CALABI'S CORRESPONDENCE

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**ABSTRACT.** Calabi [3] and Shiffman [10] observed that there exists a natural correspondence between minimal graphs in Euclidean space  $\mathbb{R}^3$  and maximal graphs in Lorentz–Minkowski space  $\mathbb{L}^3 = \mathbb{R}_1^2$  with signature  $(+, +, -)$ . See also [2, 8, 9]. Our main aim is to introduce two generalizations [6] of Calabi's correspondence.

More generally, we discover the first **twin correspondence** [5, 6] between CMC- $H$  graphs in Riemannian Bianchi–Cartan–Vranceanu space  $\mathbb{E}^3(\kappa, \tau)$  and spacelike CMC- $\tau$  graphs in Lorentzian Bianchi–Cartan–Vranceanu space  $\mathbb{L}^3(\kappa, H)$ . The following commutative diagram illustrates the twin correspondence and Daniel's sister correspondence [4] simultaneously:



When both mean curvature and bundle curvature vanish, the twin correspondence becomes Albuje-Alías duality [1] between graphs with zero mean curvature in Riemannian product space  $\mathbb{E}^3(\kappa, 0) = \mathcal{M}_\kappa \times \mathbb{R}$  and spacelike graphs with zero mean curvature in Lorentzian product space  $\mathbb{L}^3(\kappa, 0) = \mathcal{M}_\kappa \times \mathbb{R}_1$ . The first twin correspondence yields a Calabi–Chern type theorem that there exists no entire spacelike graph of zero mean curvature in Lorentzian Heisenberg group.

We construct the second **twin correspondence** [6, 7] between 2-dimensional minimal graphs in Euclidean space  $\mathbb{R}^{n+2}$  and 2-dimensional maximal graphs in pseudo-Euclidean space  $\mathbb{R}_n^{n+2}$ . The second twin correspondence induces an explicit duality for 2-variables symplectic Monge–Ampère equations (special Lagrangian equations in  $\mathbb{R}^4$  and split special Lagrangian equations in  $\mathbb{R}_2^4$ ) with prescribed Lagrangian angles.

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