TWO GENERALIZATIONS OF CALABI'S CORRESPONDENCE

Hojoo Lee (Universidad de Granada)

ABSTRACT. Calabi [3] and Shiffman [10] observed that there exists a natural correspondence between minimal graphs in Euclidean space $\mathbb{R}^3$ and maximal graphs in Lorentz–Minkowski space $\mathbb{L}^3 = \mathbb{R}^2_1$ with signature $(+,+,-)$. See also [2, 8, 9]. Our main aim is to introduce two generalizations [6] of Calabi's correspondence.

More generally, we discover the first **twin correspondence** [5, 6] between CMC-H graphs in Riemannian Bianchi–Cartan–Vranceanu space $\mathbb{E}^3(\kappa,\tau)$ and spacelike CMC-τ graphs in Lorentzian Bianchi–Cartan–Vranceanu space $\mathbb{L}^3(\kappa,H)$. The following commutative diagram illustrates the twin correspondence and Daniel's sister correspondence [4] simultaneously:

minimal graphs **in** $\mathbb{E}^3(\kappa,\tau)$		**spacelike CMC** τ**-graphs** **in** $\mathbb{L}^3(\kappa,0)$
	↻	
CMC τ**-graphs** **in** $\mathbb{E}^3(\kappa-4\tau^2,0)$		**maximal graphs** **in** $\mathbb{L}^3(\kappa-4\tau^2,\tau)$

When both mean curvature and bundle curvature vanish, the twin correspondence becomes Albujer-Alías duality [1] between graphs with zero mean curvature in Riemannian product space $\mathbb{E}^3(\kappa,0) = \mathcal{M}_\kappa \times \mathbb{R}$ and spacelike graphs with zero mean curvature in Lorentzian product space $\mathbb{L}^3(\kappa,0) = \mathcal{M}_\kappa \times \mathbb{R}_1$. The first twin correspondence yields a Calabi–Chern type theorem that there exists no entire spacelike graph of zero mean curvature in Lorentzian Heisenberg group.

We construct the second **twin correspondence** [6, 7] between 2-dimensional minimal graphs in Euclidean space $\mathbb{R}^{n+2}$ and 2-dimensional maximal graphs in pseudo-Euclidean space $\mathbb{R}^{n+2}_n$. The second twin correspondence induces an explicit duality for 2-variables symplectic Monge-Ampére equations (special Lagrangian equations in $\mathbb{R}^4$ and split special Lagrangian equations in $\mathbb{R}^4_2$) with prescribed Lagrangian angles.

References

[1] A. L. Albujer, L. J. Alías, *Calabi-Bernstein results for maximal surfaces in Lorentzian product spaces*, J. Geom. Phys. **59** (2009), 620–631.

[2] H. Araújo, M. L. Leite, *How many maximal surfaces do correspond to one minimal surface?*, Math. Proc. Camb. Phil. Soc. **146** (2009), 165–175.

[3] E. Calabi, *Examples of Bernstein problems for some non-linear equations*, Proc. Sympos. Pure Math. **15** (1970), Amer. Math. Soc., Providence, RI, 223–230.

[4] B. Daniel, *Isometric immersions into 3-dimensional homogeneous manifolds*, Comment. Math. Helv. **82** (2007), No. 1, 87–131.

[5] H. Lee, *Extension of the duality between minimal surfaces and maximal surfaces*, Geom. Dedicata, **151** (2011), 373–386.

[6] H. Lee, *Extensions of Calabi's correspondence between minimal surfaces and maximal surfaces*, http://arxiv.org/abs/1103.5180v2

[7] H. Lee, *Minimal surface systems, maximal surface systems and special Lagrangian equations*, preprint.

[8] F. J. López, R. López, R. Souam, *Maximal surfaces of Riemann type in Lorentz–Minkowski space L^3*, Michigan Math. J. **47** (2000), No. 3, 469–497.

[9] L. Mazet, M. Traizet, *A quasi-periodic minimal surface*, Comment. Math. Helv. **83** (2008), No. 3, 573–601.

[10] M. Shiffman, *On surfaces of stationary area bounded by two circles, or convex curves, in parallel planes*, Ann. of Math. **63** (1956), 77–90.