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Results on Real Hypersurfaces in $\mathbb{C}P^2$ and $\mathbb{C}H^2$

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Definition

Complex Space Form is a Kaehler manifold equipped with a complex structure J, $(J^2 = -I)$, whose holomorphic sectional curvature is constant for all the J - invariant planes Π in T_PM , for all points $P \in M$. The constant holomorphic sectional curvature is denoted by c.

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 $M_n(c), c \neq 0 \longrightarrow \text{Non-Flat Complex Space Form}$

Almost contact structure or (φ, ξ, η) - structure is a tensor field φ of type (1,1), a vector field ξ and a 1-form η , which satisfy the following relations

$$\varphi^2 X = -X + \eta(X)\xi, \quad \eta(\xi) = 1,$$

for any vector field $X \in \mathfrak{X}(M)$.

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 $(M, \varphi, \xi, \eta) \longrightarrow Almost \ contact \ manifold.$

Compatible Metric of an almost contact manifold is a Riemannian metric such that

$$\eta(X) = g(X,\xi), \ g(\varphi X,\varphi Y) = g(X,Y) - \eta(X)\eta(Y), X,Y \in \mathfrak{X}(M)$$

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Structure $(\varphi, \xi, \eta, g) \longrightarrow \textbf{Almost contact metric structure}.$ $(M, \varphi, \xi, \eta, g) \longrightarrow \textbf{Almost contact metric manifold}.$

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N: locally unit normal vector field on M.

Gauss and Weingarten equations

$$\overline{\nabla}_Y X = \nabla_Y X + g(AY, X)N,$$
$$\overline{\nabla}_X N = -AX,$$

where ∇ is the Levi-Civita connection of M, A is the shape operator of M and g the induced Riemannian metric on M.

Structure vector field ξ : $\xi = -JN$

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Metric $g: g(\varphi X, Y) = G(JX, Y)$

1-form η : $\eta(X) = g(X, \xi) = G(JX, N)$

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Tensor field \varphi of type (1,1): JX = \varphi X + \eta(X)N
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1-form $\eta : \eta(X) = g(X, \xi) = G(JX, N)$

Tensor field φ of type (1,1): $JX = \varphi X + \eta(X)N$

$$\varphi^2 X = -X + \eta(X)\xi, \quad \eta \circ \varphi = 0, \quad \varphi \xi = 0, \quad \eta(\xi) = 1,$$

$$g(\varphi X,\varphi Y)=g(X,Y)-\eta(X)\eta(Y),\ g(X,\varphi Y)=-g(\varphi X,Y),$$

$$\nabla_X \xi = \varphi A X, \qquad (\nabla_X \varphi) Y = \eta(Y) A X - g(A X, Y) \xi,$$

for any tangent vector field X, Y on M.

Gauss equation:

$$R(X,Y)Z = \frac{c}{4}[g(Y,Z)X - g(X,Z)Y + g(\varphi Y,Z)\varphi X$$
$$-g(\varphi X,Z)\varphi Y - 2g(\varphi X,Y)\varphi Z] + g(AY,Z)AX - g(AX,Z)AY$$

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Codazzi equation:

$$(\nabla_X A)Y - (\nabla_Y A)X = \frac{c}{4} [\eta(X)\varphi Y - \eta(Y)\varphi X - 2g(\varphi X, Y)\xi],$$

where R denotes the Riemannian curvature tensor on M.

$$A\xi = \alpha\xi + \beta U,$$

where $\beta = |\varphi \nabla_{\xi} \xi|$ and $U = -\frac{1}{\beta} \varphi \nabla_{\xi} \xi \in \mathbb{D}$, provided that $\beta \neq 0$.

$$A\xi = \alpha\xi + \beta U,$$

where $\beta = |\varphi \nabla_{\xi} \xi|$ and $U = -\frac{1}{\beta} \varphi \nabla_{\xi} \xi \in \mathbb{D}$, provided that $\beta \neq 0$.

Definition

A real hypersurface is a **Hopf hypersurface**, if the structure vector field ξ is principal, i.e. $A\xi = \alpha \xi$.

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where $\beta = |\varphi \nabla_{\xi} \xi|$ and $U = -\frac{1}{\beta} \varphi \nabla_{\xi} \xi \in \mathbb{D}$, provided that $\beta \neq 0$.

Definition

A real hypersurface is a **Hopf hypersurface**, if the structure vector field ξ is principal, i.e. $A\xi = \alpha \xi$.

If M is a Hopf hypersurface then α is constant

Takagi (1973) [23], Cecil, Ryan (1982) [2], Wang (1983) [25], Kimura (1986) [6]

- (A1) geodesic spheres of radius r, where $0 < r < \frac{\pi}{2}$,
- (A2) tubes of radius r over totally geodesic complex projective space $\mathbb{C}P^k$, where $0 < r < \frac{\pi}{2}$ and $1 \le k \le n-2$,
- (B) tubes of radius r over complex quadrics and $\mathbb{R}P^n$, where $0 < r < \frac{\pi}{4}$,
- (C) tubes of radius r over the Serge embedding of $\mathbb{C}P^1 \times \mathbb{C}P^{\frac{(n-1)}{2}}$, where $0 < r < \frac{\pi}{4}$ and $n \ge 2\kappa + 3$, $\kappa \in \mathbb{N}^*$,

- (D) tubes of radius r over the *Plucker* embedding of the complex Grassmannian manifold $G_{2,5}$, where $0 < r < \frac{\pi}{4}$ and n = 9.
- (E) tubes of radius r over the canonical embedding of the Hermitian symmetric space SO(10)/U(5), where $0 < r < \frac{\pi}{4}$ and n = 15.

Complex Hyperbolic Space $\mathbb{C}H^n, n \geq 2$

Montiel (1985) [7], Berndt (1989) [1]

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- (A0) horospheres
- (A1,0) geodesic spheres of radius r > 0,
- (A1,1) tubes of radius r > 0 over totally geodesic complex hyperbolic hyperplanes $\mathbb{C}H^{n-1}$,
- (A2) tubes of radius r > 0 over totally geodesic submanifold $\mathbb{C}H^k$, $1 \le k \le n-2$,
- (B) tubes of radius r > 0 over totally real hyperbolic space $\mathbb{R}H^n$.

Case of $\mathbb{C}P^2$

A three dimensional real hypersurface M is locally congruent to

- (A1) a geodesic sphere of radius r, where $0 < r < \frac{\pi}{2}$,
- (B) a tube of radius r over complex quadrics and $\mathbb{R}P^n$, where $0 < r < \frac{\pi}{4}$.

Type	α	λ	ν	m_{α}	m_{λ}	$m_{ u}$
A1	$2\cot 2r$	$\cot r$	-	1	2	-
В	$2\cot 2r$	$\cot(r - \frac{\pi}{4})$	$-\tan(r-\frac{\pi}{4})$	1	1	1

Case of $\mathbb{C}H^{2^{l}}$

A three dimensional real hypersurface M is locally congruent to

- (A0) a horosphere
- (A1,0) a geodesic sphere of radius r > 0,
- (A1,1) a tube of radius r > 0 over totally geodesic complex hyperbolic hyperplanes $\mathbb{C}H^1$,
- (B) a tube of radius r > 0 over totally real hyperbolic space $\mathbb{R}H^2$.

Type	α	λ	ν	m_{lpha}	m_{λ}	$m_{ u}$
A0	2	1	-	1	2	-
A1,0	$2\coth(2r)$	$\coth(r)$	-	1	2	-
A1,1	$2\coth(2r)$	tanh(r)	-	1	2	-
В	$2\tanh(2r)$	tanh(r)	$\coth(r)$	1	1	1

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Case of
$$\mathbb{C}P^n$$
, $n \geq 2$, \longrightarrow Okumura, [9]

Case of
$$\mathbb{C}H^n$$
, $n \geq 2 \longrightarrow$ Montiel-Romero [8]

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Case of
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, $n \geq 2 \longrightarrow \text{Montiel-Romero}$ [8]

Theorem

Let M be a real hypersurface in $M_n(c)$, $n \ge 2$, then $\varphi A = A\varphi$ if and only if M is an open subset of real hypersurfaces of type A.

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Structure Jacobi Operator

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Definition

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Definition

Structure Jacobi operator of real hypersurface is defined by the relation $l = R(\cdot, \xi)\xi$.

$$lX = \frac{c}{4}[X - \eta(X)\xi] + \alpha AX - \eta(AX)A\xi$$

Covariant Derivative of Structure Jacobi Operator

Parallel



Covariant Derivative of Structure Jacobi Operator

Parallel

$$\begin{array}{l}
\downarrow \\
(\nabla_X l)Y = 0, \\
X \in TM
\end{array}$$

Covariant Derivative of Structure Jacobi Operator Parallel D-Parallel

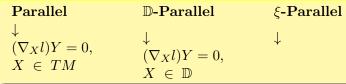


$$(\nabla_X l)Y = 0, X \in TM$$

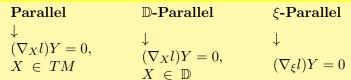
Covariant Derivative of Structure Jacobi Operator

$\begin{array}{ll} \mathbf{Parallel} & & \mathbb{D}\text{-}\mathbf{Parallel} \\ \downarrow & & \downarrow \\ (\nabla_X l) Y = 0, & & \downarrow \\ X \in TM & & (\nabla_X l) Y = 0, \\ X \in \mathbb{D} & & \end{array}$

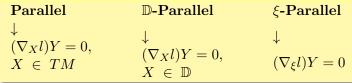
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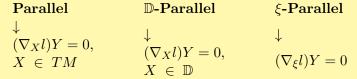
Lie Derivative of Structure Jacobi Operator

Lie

Parallel

.1

Covariant Derivative of Structure Jacobi Operator



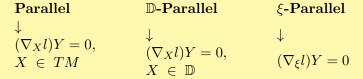
Lie Derivative of Structure Jacobi Operator

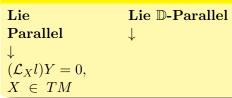
Lie

Parallel

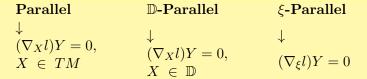
$$\downarrow \\
(\mathcal{L}_X l) Y = 0, \\
X \in TM$$

Covariant Derivative of Structure Jacobi Operator



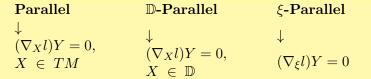


Covariant Derivative of Structure Jacobi Operator



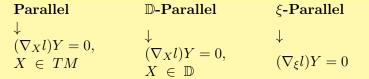
$$\begin{array}{lll} \textbf{Lie} & \textbf{Lie} \; \mathbb{D}\textbf{-Parallel} \\ \textbf{Parallel} & \downarrow & \\ \downarrow & (\mathcal{L}_X l) Y = 0, \\ (\mathcal{L}_X l) Y = 0, & X \in \mathbb{D} \\ X \in TM \end{array}$$

Covariant Derivative of Structure Jacobi Operator



Lie	$\operatorname{Lie} \mathbb{D} ext{-}\operatorname{Parallel}$	Lie ξ -Parallel
Parallel	\downarrow	I
\downarrow	$(\mathcal{L}_X l)Y = 0,$	+
$(\mathcal{L}_X l)Y = 0,$	$X \in \mathbb{D}$	
$X \in TM$		

Covariant Derivative of Structure Jacobi Operator



Lie	$\mathbf{Lie} \ \mathbb{D}\text{-}\mathbf{Parallel}$	Lie ξ -Parallel
Parallel	\downarrow	1
\downarrow	$(\mathcal{L}_X l)Y = 0,$	+
$(\mathcal{L}_X l)Y = 0,$	$X \in \mathbb{D}$	$(\mathcal{L}_{\xi}l)Y = 0$
$X \in TM$		

Ortega, Perez and Santos (2006) [10]: Non-existence of real hypersurfaces in complex space form $M_n(c)$, $n \geq 2$, whose structure Jacobi operator is parallel.

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QUESTION

Ortega, Perez and Santos (2006) [10]: Non-existence of real hypersurfaces in complex space form $M_n(c)$, $n \geq 2$, whose structure Jacobi operator is parallel.

Perez, Santos and Suh (2006) [18]: Non-existence of real hypersurfaces in $\mathbb{C}P^n$, $n \geq 3$ equipped with \mathbb{D} -parallel structure Jacobi operator.

QUESTION

Do there exist real hypersurfaces equipped with ξ -parallel structure Jacobi operator?

Theorem [14]

Let M be a real hypersurface in $M_2(c)$, whose structure Jacobi operator is ξ -parallel. Then

- in case of $\mathbb{C}P^2$, M is locally congruent to either a geodesic sphere of radius r, where $0 < r < \frac{\pi}{2}$, or a non-homogeneous real hypersurface, which is considered as a tube of radius $r = \frac{\pi}{4}$ over a holomorphic curve in $\mathbb{C}P^2$
- in case of $\mathbb{C}H^2$, M is locally congruent to a horosphere, or to a geodesic sphere, or to a tube over $\mathbb{C}H^1$, or to a Hopf hypersurface with $A\xi = 0$.

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Perez and Santos, (2005) [16]: Non-existence of real hypersurfaces in $\mathbb{C}P^n$, $n \geq 3$, equipped with Lie-parallel structure Jacobi operator, i.e. $(\mathcal{L}_X l)Y = 0$, with $X, Y \in TM$.

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- 1) Classification of three-dimensional real hyperusurfaces in $M_2(c)$ with Lie ξ -parallel structure Jacobi operator.
- 2) Non-existence of real hypersurfaces in $M_n(c)$, $n \geq 2$, with Lie-parallel structure Jacobi operator.

Conditions on the Lie derivative of the structure Jacobi

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Theorem [-, Xenos 2012] [12]

There are no real hypersurfaces in $M_2(c)$ equipped with Lie \mathbb{D} -parallel structure Jacobi operator.

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Do there exist real hypersurfaces whose structure Jacobi operator is Lie \mathbb{D} -parallel?

Theorem [-, Xenos 2012] [12]

There are no real hypersurfaces in $M_2(c)$ equipped with Lie \mathbb{D} -parallel structure Jacobi operator.

Theorem [Perez and Suh] [22]

There do not exist Hopf real hypersurfaces in $\mathbb{C}P^n$, $n \geq 3$, equipped with Lie \mathbb{D} -parallel structure Jacobi operator.

Pseudo-parallel Structure Jacobi Operator

Definition

A tensor field P of type (1,s) is **semi-parallel**, if $R \cdot P = 0$.

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Definition

A tensor field P of type (1,s) is **pseudo-parallel**, if there exists a function L such that $R \cdot P = L\{(X \wedge Y) \cdot P\}$, where $(X \wedge Y)Z = g(Y,Z)X - g(X,Z)Y$.

Definition

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Definition

A tensor field P of type (1,s) is **pseudo-parallel**, if there exists a function L such that $R \cdot P = L\{(X \wedge Y) \cdot P\}$, where $(X \wedge Y)Z = g(Y,Z)X - g(X,Z)Y$.

If
$$L \neq 0 \longrightarrow \mathbf{proper}$$

Definition

A tensor field P of type (1,s) is **semi-parallel**, if $R \cdot P = 0$.

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A tensor field P of type (1,s) is **pseudo-parallel**, if there exists a function L such that $R \cdot P = L\{(X \wedge Y) \cdot P\}$, where $(X \wedge Y)Z = g(Y,Z)X - g(X,Z)Y$.

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$$(R(X,Y) \cdot P)(X_1, ..., X_s) = R(X,Y)(P(X_1, ..., X_s) - \sum_{j=1}^s P(X_1, ..., R(X,Y)X_j, ..., X_s).$$

Cho and Kimura (2010) [3]: Non-existence of real hypersurfaces in $\mathbb{C}P^n$ and $\mathbb{C}H^n$ equipped with semi-parallel structure Jacobi operator.

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$$R(X,Y)lZ - l(R(X,Y)Z)) = L\{g(lZ,Y)X - g(lZ,X)Y - l(g(Z,Y)X) + l(g(Z,X)Y)\}$$

Theorem [-, Xenos 2012] [13]

Let M be a real hypersurface in $M_2(c)$, whose structure Jacobi operator is pseudo-parallel. Then

- in case of $\mathbb{C}P^2$, M is locally congruent to either a geodesic sphere of radius r, where $0 < r < \frac{\pi}{2}$, or a non-homogeneous real hypersurface, which is considered as a tube of radius $r = \frac{\pi}{4}$ over a holomorphic curve in $\mathbb{C}P^2$
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Relation $\mathcal{L}_{\xi}l = \nabla_{\xi}l$

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Theorem

Let M be a real hypersurface in $\mathbb{C}P^n$, $n \geq 3$, whose structure Jacobi operator satisfies $\mathcal{L}_{\xi}l = \nabla_{\xi}l$. Then M is locally congruent:

- either to a tube of radius $\frac{\pi}{4}$ over a complex submanifold of $\mathbb{C}P^n$.
- or a tube of radius $r \neq \frac{\pi}{4}$ over $\mathbb{C}P^k$, where $0 \leq k \leq n-1$.

Theorem [-,Xenos 2012] [11]

Let M be a real hypersurface in $M_2(c)$, whose structure Jacobi operator satisfies relation $\mathcal{L}_{\xi}l = \nabla_{\xi}l$. Then

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Theorem [15]

There do not exist real hypersurfaces in $M_2(c)$, whose structure Jacobi operator satisfies the above relation.

Lie recurrence

Definition

A tensor field P of type (1,1) is called **recurrent**, if there exists a 1-form ω such that $(\nabla_X P) = \omega(X)P(Y)$, where X, Y are tangent to M.

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Theofanidis and Xenos (2012) [24]: Non-existence of real hypersurfaces in $M_n(c)$, $n \geq 3$, whose structure Jacobi operator is recurrent.

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Theorem [5]

There do not exist real hypersurfaces in $M_2(c)$, whose structure Jacobi operator is Lie recurrent.

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 - Conditions on the Lie derivative of the structure Jacobi operator
- Sketch of Proof
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We consider a local orthonormal basis $\{U, \varphi U, \xi\}$.

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 $\mathcal{V} \cup \Omega$ is open and dense in the closure of \mathcal{N} .

Lemma 1

Let M be a non-Hopf real hypersurface in $M_2(c)$. Then the following relations hold on M

$$AU = \gamma U + \delta \varphi U + \beta \xi, \quad A\varphi U = \delta U + \mu \varphi U, \quad A\xi = \alpha \xi + \beta U,$$

$$\nabla_U \xi = -\delta U + \gamma \varphi U, \quad \nabla_{\varphi U} \xi = -\mu U + \delta \varphi U, \quad \nabla_{\xi} \xi = \beta \varphi U,$$

$$\nabla_U U = \kappa_1 \varphi U + \delta \xi, \ \nabla_{\varphi U} U = \kappa_2 \varphi U + \mu \xi, \nabla_{\xi} U = \kappa_3 \varphi U,$$

$$\nabla_U \varphi U = -\kappa_1 U - \gamma \xi, \nabla_{\varphi U} \varphi U = -\kappa_2 U - \delta \xi, \nabla_{\xi} \varphi U = -\kappa_3 U - \beta \xi$$

where $\gamma, \delta, \mu, \kappa_1, \kappa_2, \kappa_3$ are smooth functions on M.

Lemma 2

Let M be a non-Hopf real hypersurface in $M_2(c)$. Then the following relations hold on M

$$\begin{split} U\beta - \xi\gamma &= \alpha\delta - 2\delta\kappa_3 \\ \xi\delta &= \alpha\gamma + \beta\kappa_1 + \delta^2 + \mu\kappa_3 + \frac{c}{4} - \gamma\mu - \gamma\kappa_3 - \beta^2 \\ U\alpha - \xi\beta &= -3\beta\delta \\ \xi\mu &= \alpha\delta + \beta\kappa_2 - 2\delta\kappa_3 \\ (\varphi U)\alpha &= \alpha\beta + \beta\kappa_3 - 3\beta\mu \\ (\varphi U)\beta &= \alpha\gamma + \beta\kappa_1 + 2\delta^2 + \frac{c}{2} - 2\gamma\mu + \alpha\mu \\ U\delta - (\varphi U)\gamma &= \mu\kappa_1 - \kappa_1\gamma - \beta\gamma - 2\delta\kappa_2 - 2\beta\mu \\ U\mu - (\varphi U)\delta &= \gamma\kappa_2 + \beta\delta - \kappa_2\mu - 2\delta\kappa_1 \end{split}$$

2^{nd} Step

- 1) Hopf Hypersurface $\longrightarrow A\xi = \alpha\xi \longrightarrow \alpha = \text{constant}.$
- 2) We consider a point $P \in M$ and we choose principal vector field $Z \in \ker(\eta)$ at P such that m

$$AZ = \lambda Z$$
 and $A\varphi Z = \nu \varphi Z$,

 λ , ν functions

$$\lambda \nu = \frac{\alpha}{2}(\lambda + \nu) + \frac{c}{4}$$

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- - Three-dimensional Real Hypersurfaces in $\mathbb{C}P^2$ and $\mathbb{C}H^2$
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Condition	$\mathbb{C}P^2$ and $\mathbb{C}H^2$	$\mathbb{C}P^n, n \geq 3$	$\mathbb{C}H^n, n \ge 3$
ξ-Parallel	V	?	?
Lie $\mathbb D$ - parallel	No	No	?
Pseudo - parallel	V	?	?

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Lie $\mathbb D$ - parallel	No	No	?
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$\mathcal{L}_{\xi}l= abla_{\xi}l$	V	V	?
$\mathcal{L}_X l = \nabla_X l, X \in \mathbb{D}$	No	?	?
Lie recurrence	No	No	No

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Lie recurrence	No	No	No

References I



1. J. Berndt, "Real hypersurfaces with constant principal curvatures in complex hyperbolic space", J. Reine Angew. Math., 395 (1989), 132-141.



 T. Cecil and P. J. Ryan, "Focal sets and real hypersurfaces in complex projective spaces", Trans. Amer. Math. Soc., 269 (1982), 481-499.



3. J. T. Cho and M. Kimura, "Curvature of Hopf Hypersurfaces in a Complex Space form", Results Math., (in electronic form)



4. T. A. Ivey and P. J. Ryan, "The structure Jacobi operator for real hypersurfaces in CP² and CH²", Result. Math., 56 (2009), 473-488.



5. G. Kaimakamis and K. Panagiotidou, "Real hypersurfaces in non-flat complex space form with Lie recurrent structure Jacobi operator", submitted.



 M. Kimura, "Real Hypersurfaces and Complex Submanifolds in complex projective spaces," Trans. Amer. Math. Soc., 296 (1986), 137-149.



7. S. Montiel, "Real hypersurfaces of a complex hyperbolic space", J. Math. Soc. Japan, 35 (1985), 515-535.



 S. Montiel and A. Romero, "On some real hypersurfaces of a complex hyperbolic space", Geom. Dedicta, 20 (1986), no. 2, 245-261.

References II



9. M. Okumura, "On some real hypersurfaces of a complex projective space", *Trans. Amer. Math. Soc.*, **212** (1975), 355-364.



10. M. Ortega, J. D. Perez and F. G. Santos, "Non-existence of real hypersurfaces with parallel structure Jacobi operator in nonflat complex space forms", *Rocky Mountain J. Math.*, **36** (2006), no. 5, 1603-1613.



11. K. Panagiotidou and Ph. J. Xenos, "Real hypersurfaces in $\mathbb{C}P^2$ and $\mathbb{C}H^2$ equipped with structure Jacobi operator satisfying $\mathcal{L}_{\xi}l = \nabla_{\xi}l$ ", Advances in Pure Mathematics, 2 (2012), no. 1, 1-5.



12. K. Panagiotidou and Ph. J. Xenos, "Real hypersurfaces in $\mathbb{C}P^2$ and $\mathbb{C}H^2$ whose structure Jacobi operator is Lie \mathbb{D} -parallel", to appear Note di Matematica.



13. K. Panagiotidou and Ph. J. Xenos, "Real hypersurfaces equipped with pseudo-parallel structure Jacobi operator in $\mathbb{C}P^2$ and $\mathbb{C}H^2$ ", to appear in Houston Journal of Mathematics.



14. K. Panagiotidou and Ph. J. Xenos, "Real hypersurfaces equipped with ξ -parallel structure Jacobi operator in $\mathbb{C}P^2$ and $\mathbb{C}H^2$ ", arxiv:1201.2910.



15. K. Panagiotidou, "The structure Jacobi operator and the shape operator of real hypersurfaces in $\mathbb{C}P^2$ and $\mathbb{C}H^2$ ", arxiv:1209.0131.

References III



 J. D. Perez and F. G. Santos, "On the Lie derivative of structure Jacobi operator of real hypersurfaces in complex projective space", *Publ. Math. Debrecen*, 66 (2005), 269-282.



17. J. D. Perez, F. G. Santos and Y. J. Suh, "Real hypersurfaces in complex projective space whose structure Jacobi operator is Lie ξ -parallel", *Differential Geom. Appl.*, **22** (2005), no. 2, 181-188.



18. J. D. Perez, F. G. Santos and Y. J. Suh, "Real hypersurfaces in complex projective space whose structure Jacobi operator is D-parallel", *Bull. Belg. Math. Soc. Simon Stevin*, **13** (2006), no. 3, 459-469.



19. J. D. Perez and F. G. Santos, "Real hypersurfaces in complex projective space with recurrent structure Jacobi operator", Diff. Geom. Appl., 26 (2008), 218-223/



 J. D. Perez and F. G. Santos, "Real hypersurfaces in Complex Projective Space Whose Structure Jacobi Operator is Cyclic-Ryan Parallel", Kyungpook Math. J., 49 (2009), 211-219.



21. J. D. Perez and F. G. Santos, "Real hypersurfaces in complex projective space whose structure Jacobi operator satisfies $\mathcal{L}_{\xi}R_{\xi}=\nabla_{\xi}R_{\xi}$ ", Rocky Mountain J. Math., 39.



22. J. D. Perez and Y. J. Suh, "Real hypersurfaces in complex projective space whose structure Jacobi operator is Lie $\mathbb D$ - parallel , Canad. Math. Bull., 39.

References IV



23. R. Takagi, "Real hypersurfacesin a complex prjective space with constant principal curvatures", J. Math. Soc. Japan, 27 (1975), 43-53.



24. Th. Theofanidis and Ph. J. Xenos, "Non-existence of real hypersurfaces equipped with recurrent structure Jacobi operator in non-flat complex space forms, *Results Math.*, **61** (2012), 43-55.



25. Q. M. Wang, "Real hypersurfaces with constant principal curvatures in complex projective spaces (I), Sci. Sinica. Ser. A, 26 (1983), 1017-1024.

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