# Foliations with special geometric properties

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## Introduction

The theory of foliation is studied mainly in four aspects: analytical, topological, geometrical and dynamical. We shall concentrate on the geometry of foliations and partial answers on general question: which geometrical properties do (not) allow to foliate a given manifold by leaves having these properties.

## 1 Geometry of foliations

We start with a very elementary introduction to foliations including classical results and constructions and recall some extrinsic properties of leaves coming from Riemannian metric. Then we list results of non–existence of foliations on compact hyperbolic manifolds with totally geodesic leaves (or leaves not far from totally geodesic).

## 2 Foliations in hyperbolic spaces

As a second step, we shall develop real and complex hyperbolic geometry to study foliations of Hadamard manifolds with all leaves being Hadamard. Especially in case of real hyperbolic space $\mathbb{H}^n$, we construct the canonical embedding of leaf ideal boundaries into ideal boundary of the carrying space. Some typical examples and classification of totally geodesic foliations of $\mathbb{H}^n$ will be provided.

## 3 Conformal approach

Final part will be devoted to conformal geometry with applications to foliations. We represent codimension 1 spheres in $S^n$ as points in the quadric $\Lambda^{n+1}$ in the

Lorentz space $\mathbb{R}^{n+2}$. On the other hand, we study conformal invariants of curves and surfaces in $S^3$. These methods allow to prove non–existence of umbilical (resp. constant conformal invariant) foliation on compact hyperbolic manifolds and classify umbilical foliations of $\mathbb{H}^n$.

## References


[1] M. Badura, M. Czarnecki, *Recent progress in geometric foliation theory*, in Foliations 2012, World Scientific 2013, 9–21.

[2] A. Candel, L. Conlon, *Foliations I*, AMS 2000.

[3] M. Czarnecki, *Hadamard foliations of* $\mathbb{H}^n$, Diff. Geom. Appl. **20** (2004), 357–365.

[4] M. Czarnecki, P. Walczak, *Extrinsic geometry of foliations*, Foliations 2005, World Scientific 2006, 149–167.

[5] W. Goldman, *Complex Hyperbolic Geometry*, Oxford University Press 1999.

[6] R. Langevin, P. Walczak, *Conformal geometry of foliations*, Geom. Dedicata **132** (2008), 135–178.

[7] J. O'Hara, *Energy of Knots and Conformal Geometry*, World Scientific 2003.