STATIC & STATIONARY:

CAUSAL CURIOSITIES &

FUNDAMENTAL COCYCLE

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Lorentz-signature metric on $\mathbb{R}^n$

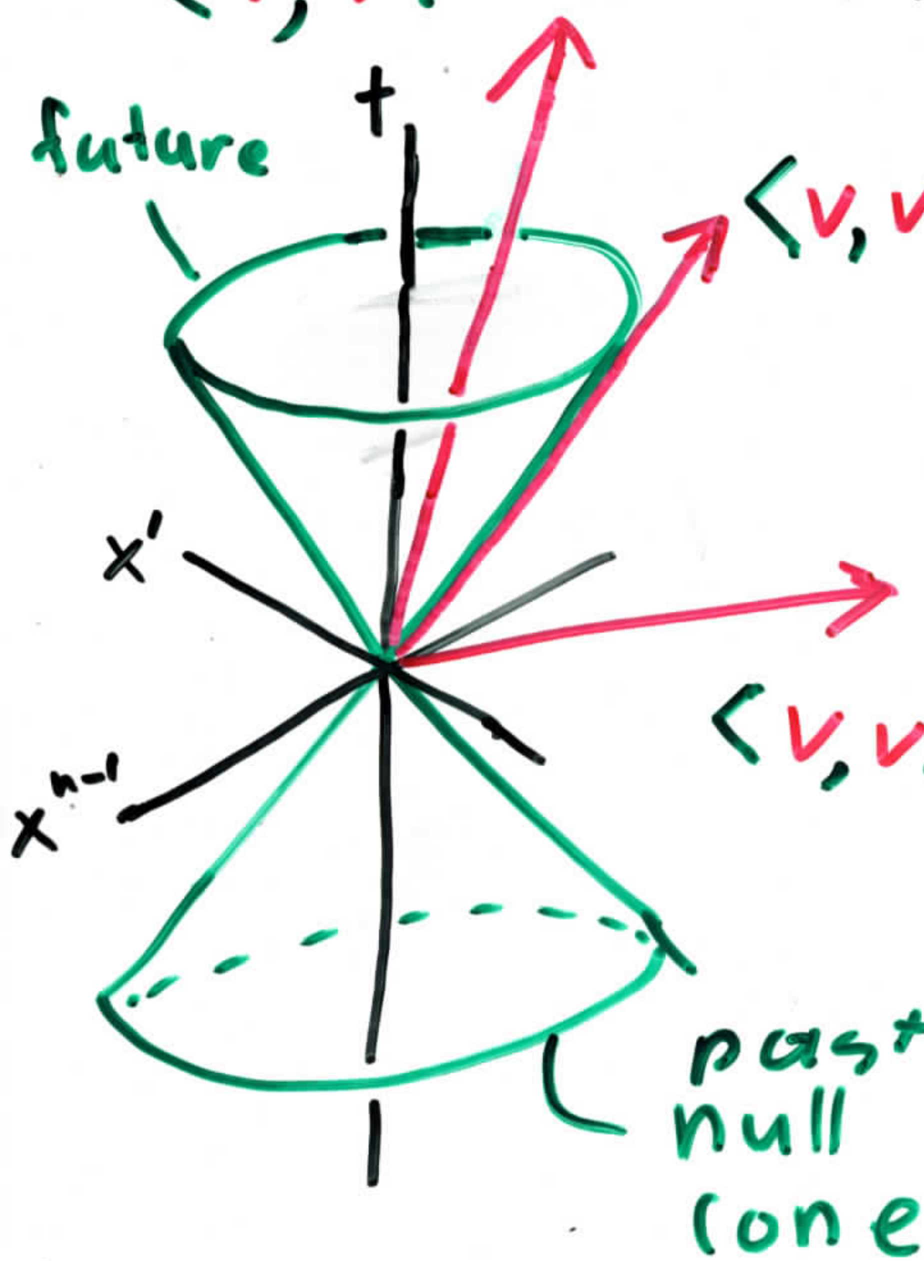
$$-dt^2+(dx^1)^2+\cdots(dx^{n-1})^2$$

$$\langle v,w\rangle = -v^0w^0+v^1w^1+\cdots+v^{n-1}w^{n-1}$$

$\langle v,v\rangle<0$ timelike

$\langle v,v\rangle=0$ null

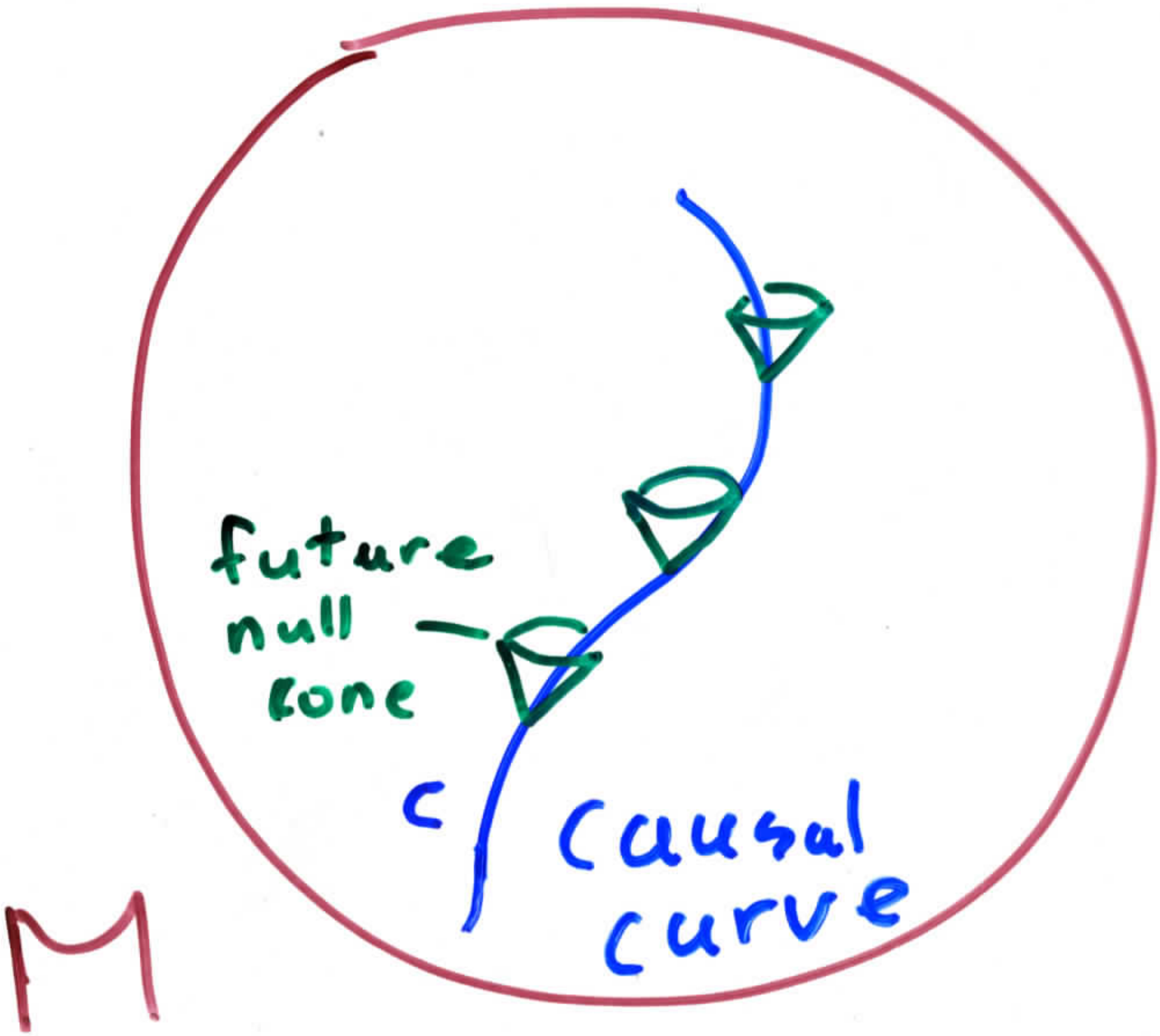
causal



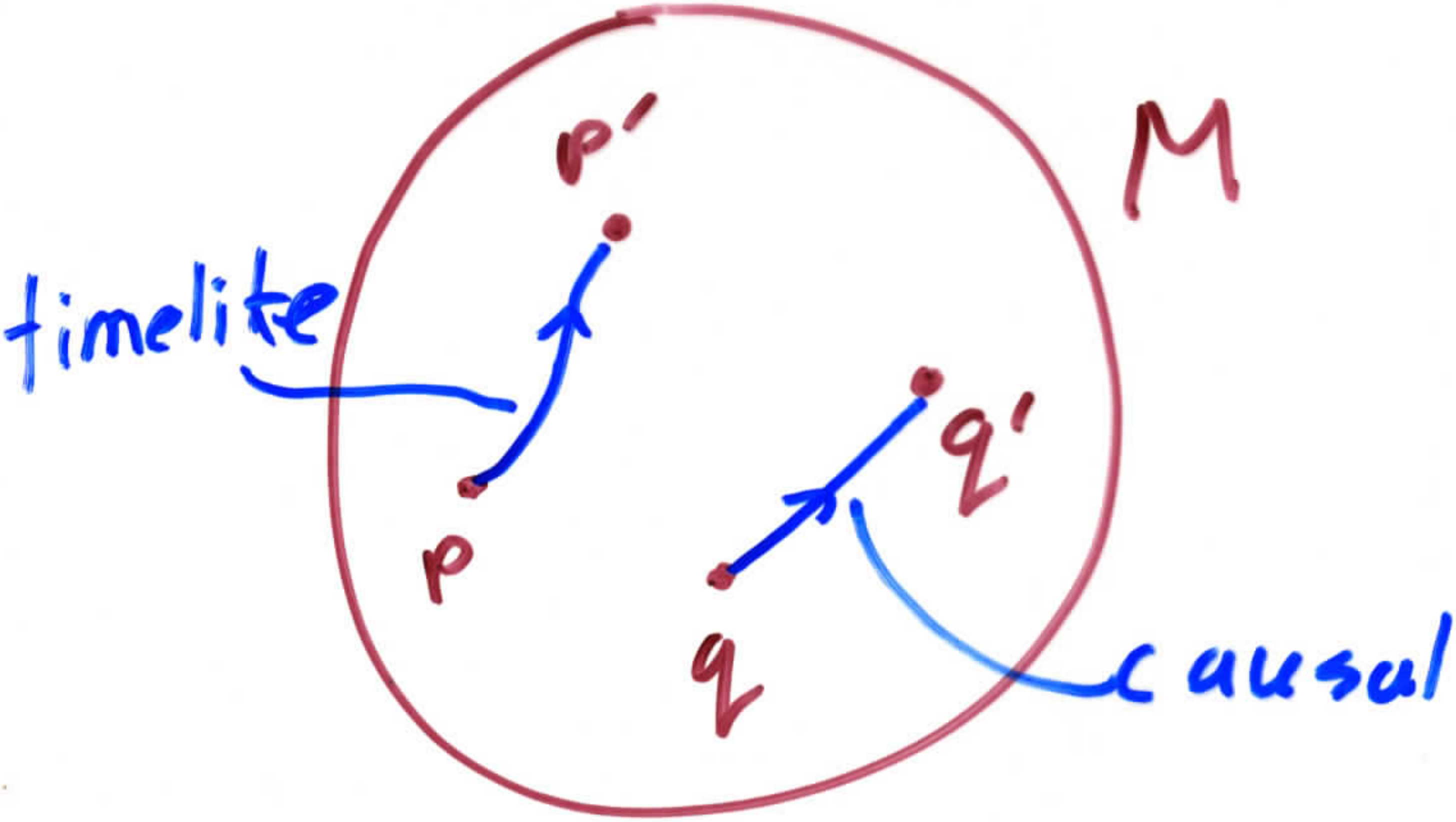
$\langle v,v\rangle>0$ spacelike

spacetime M^n

manifold w/ Lorentz metric in each T_pM



$\langle \dot{c}, \dot{c} \rangle \leq 0$



$p \ll p'$ $\exists$ future-timelike curve $p \to p'$

(chronologically precedes)

(massive particles)

$q \leq q'$ $\exists$ future-causal curve $q \to q'$

(causally precedes)

(photons)

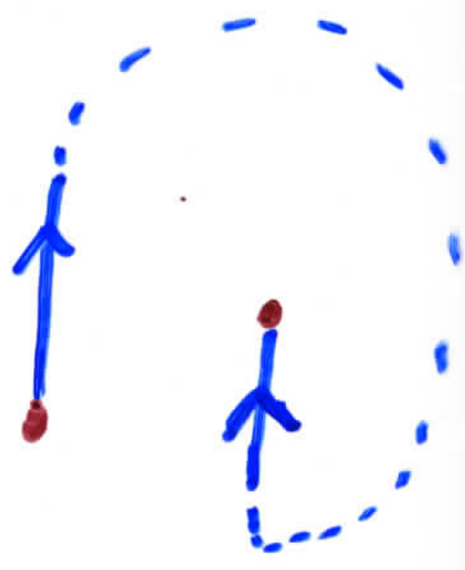
M is

chronological:
no closed timelike curves

causal:
no closed causal curves

strongly causal:
local causality
= global causality

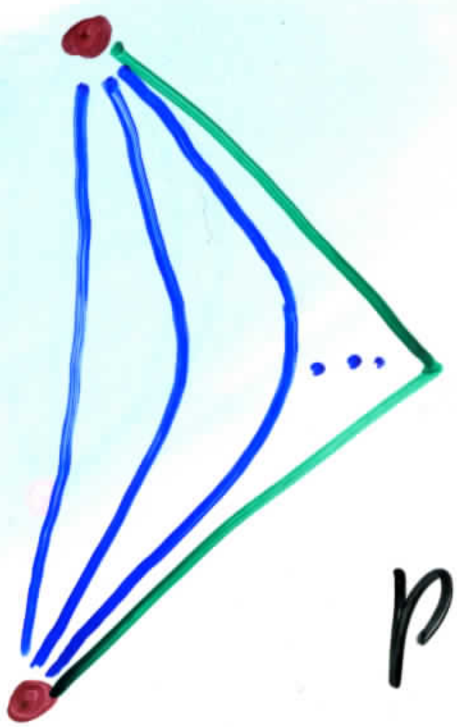
no



M is

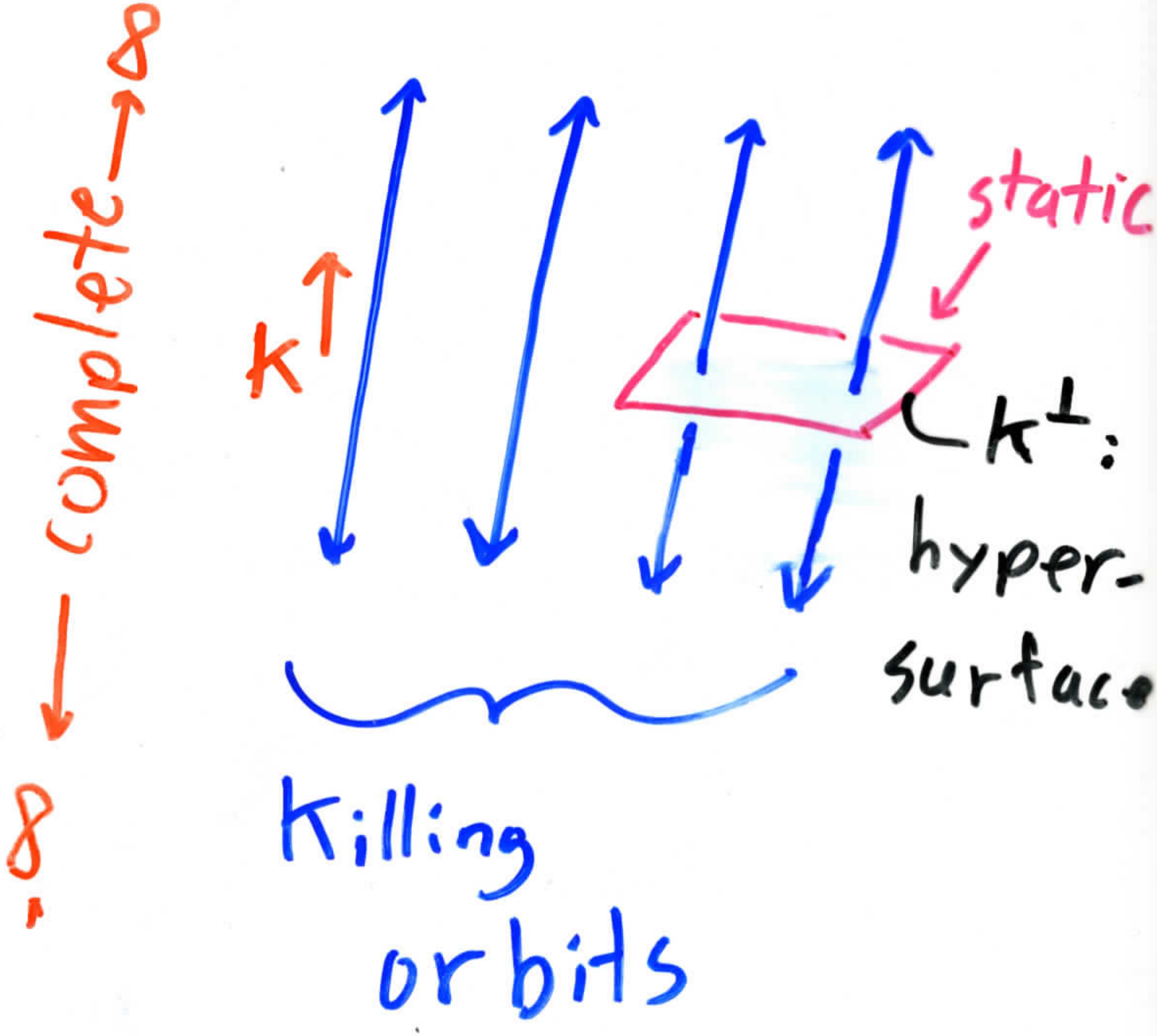
globally hyperbolic:

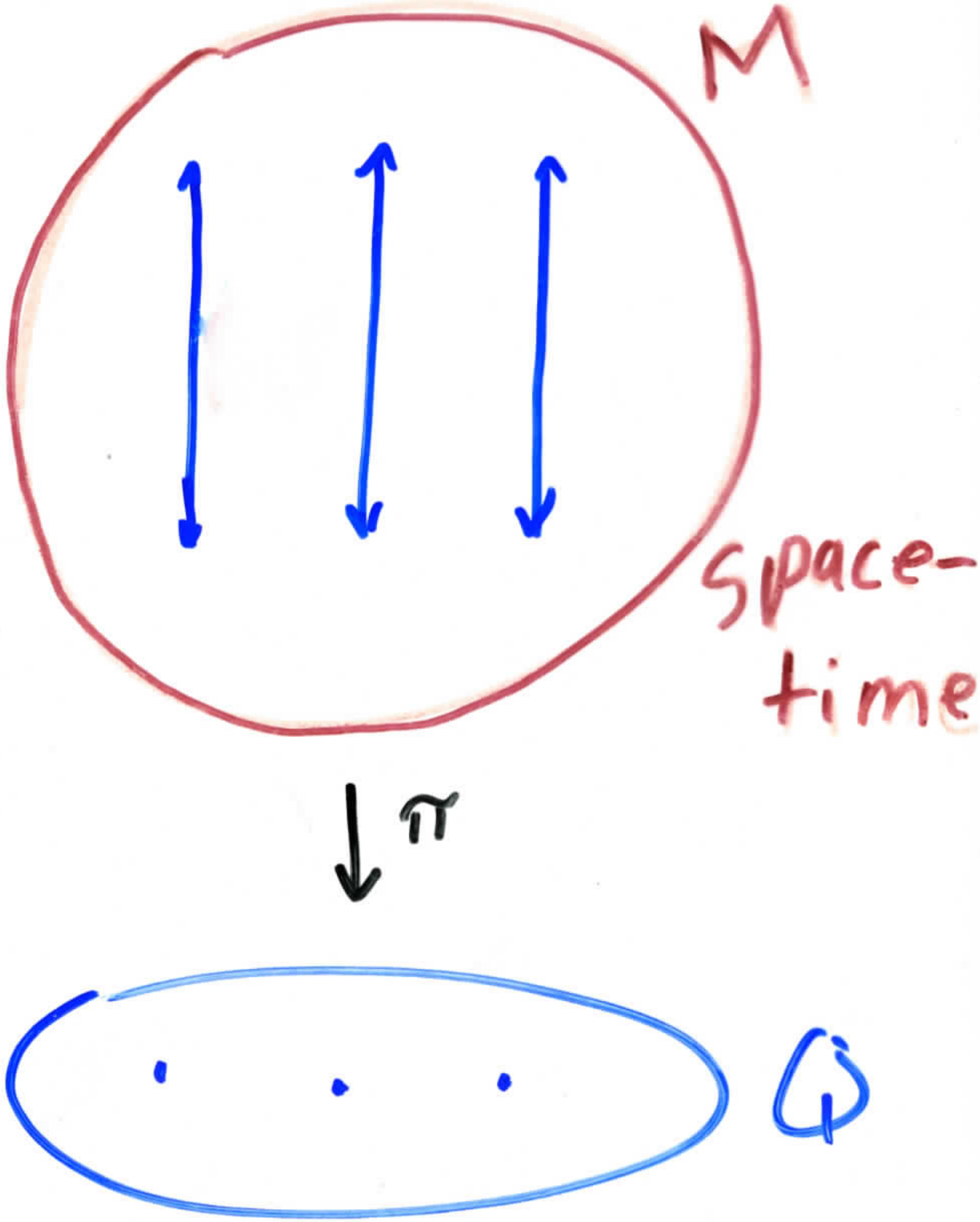
strongly causal and
compactness of
causal curves



physical
predictability!

stationary:
timelike Killing field K

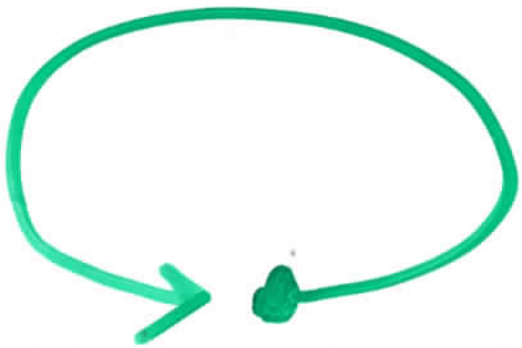


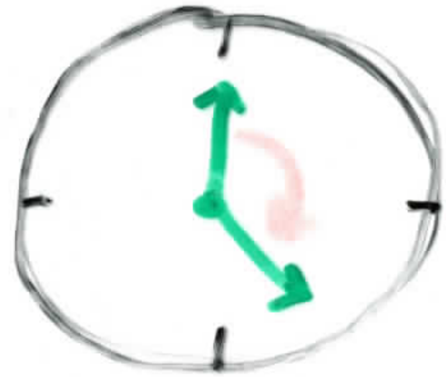


space of stationary orbits

causal curiosity:

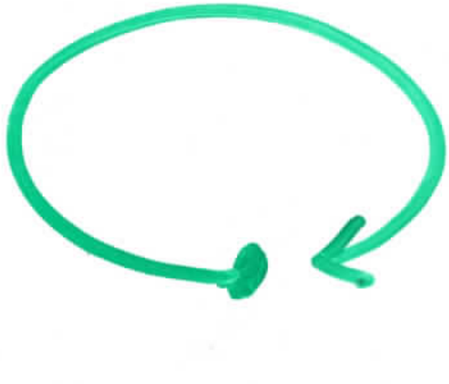
photon track:

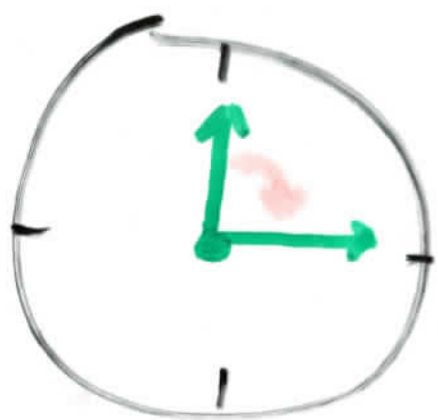




T_1

track-length $= L$

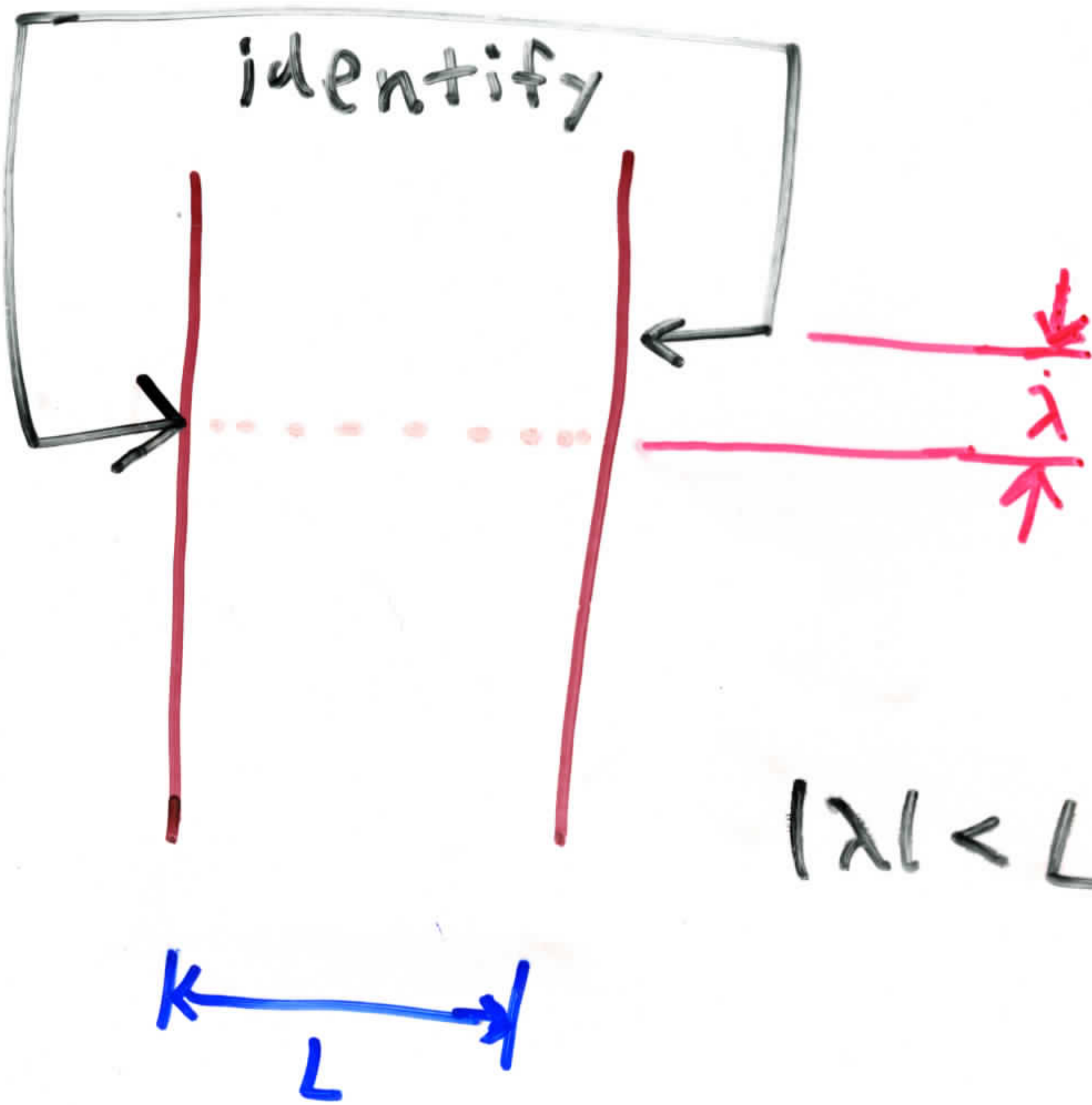




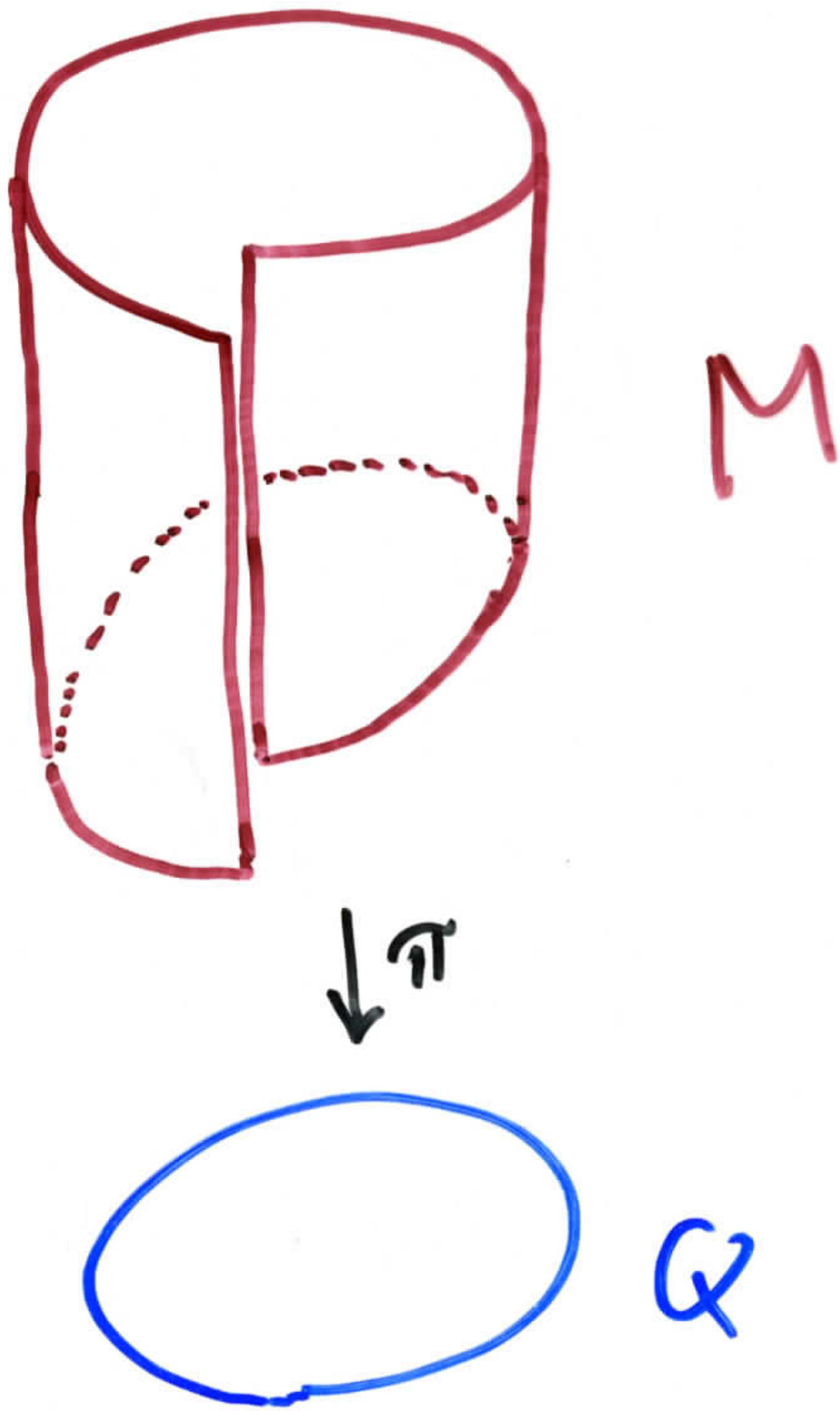
T_2

$T_1 \neq T_2$

1+1 Minkowski cylinder

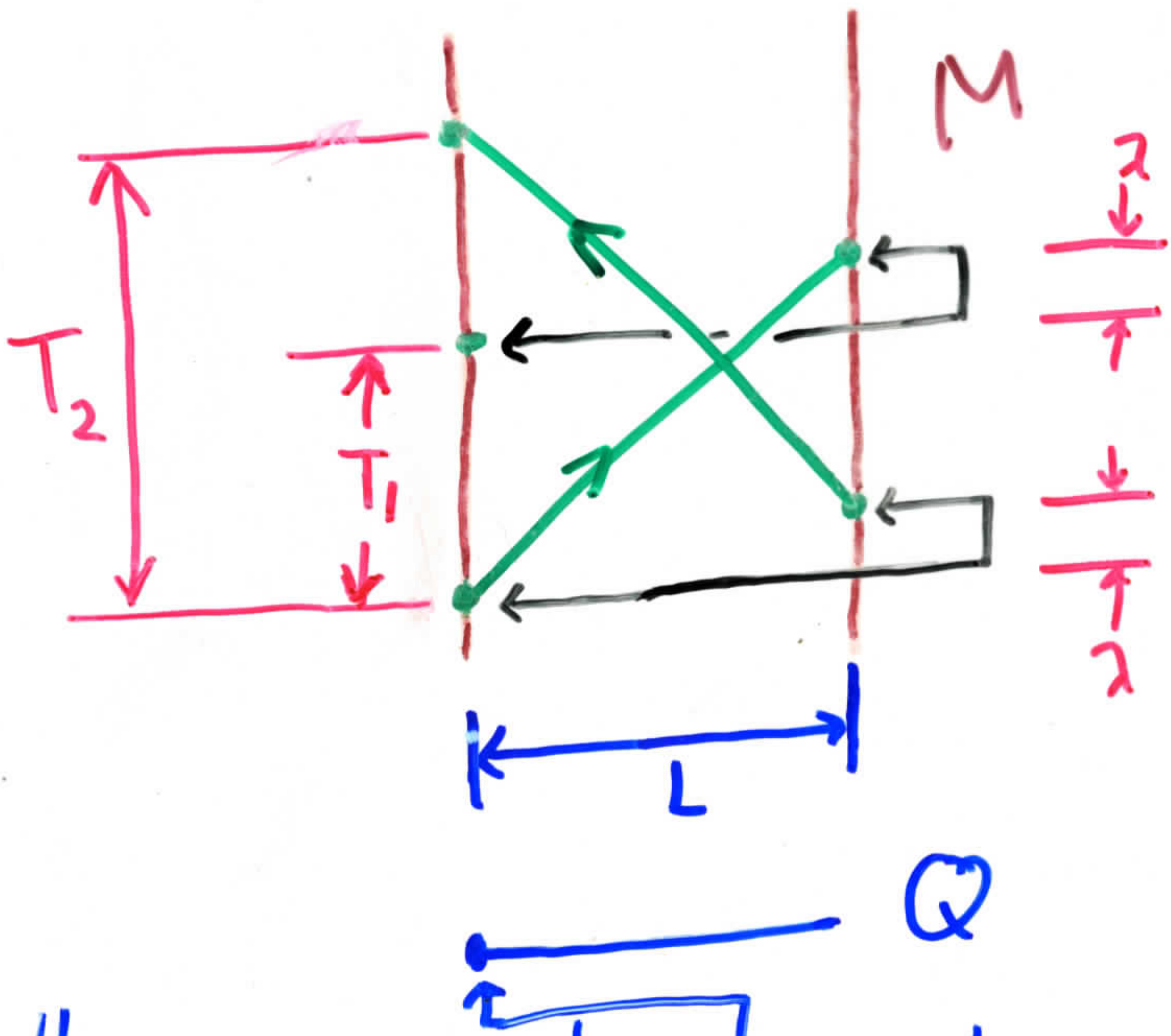


$|\lambda| < L$


M
π
Q

$$T_1 = L - \lambda$$

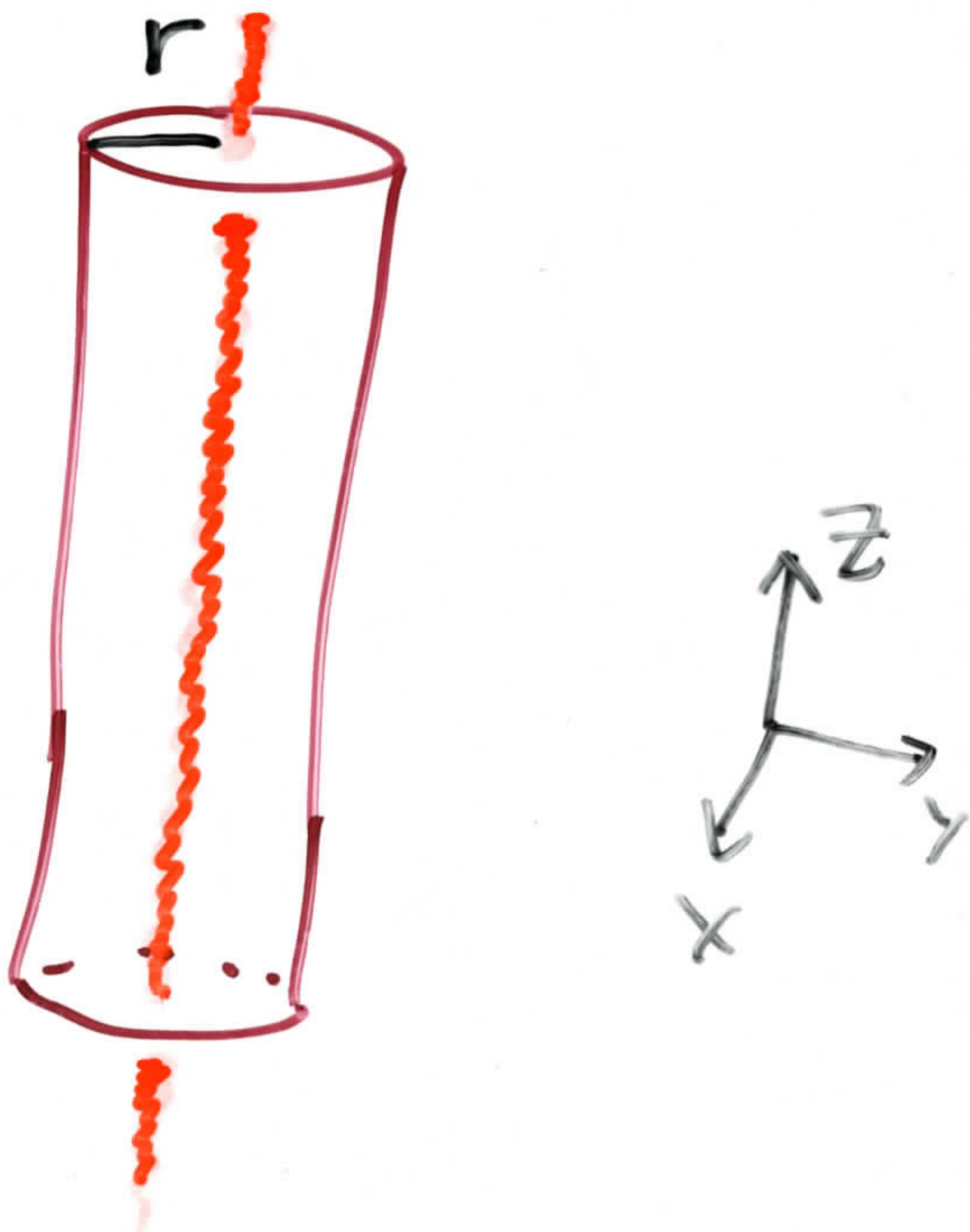
$$T_2 = L + \lambda$$



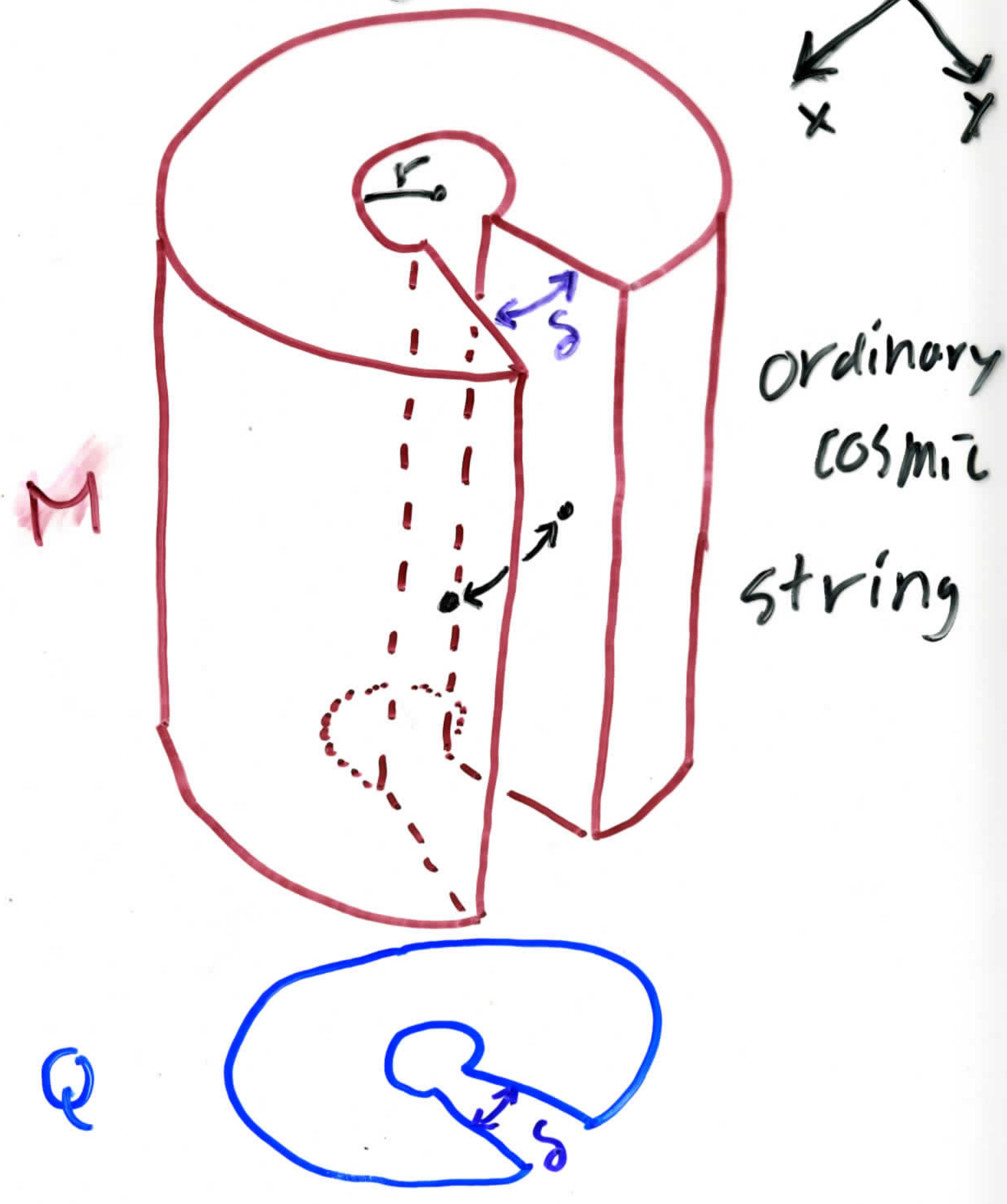
all measured by one observer

COSMIC STRING

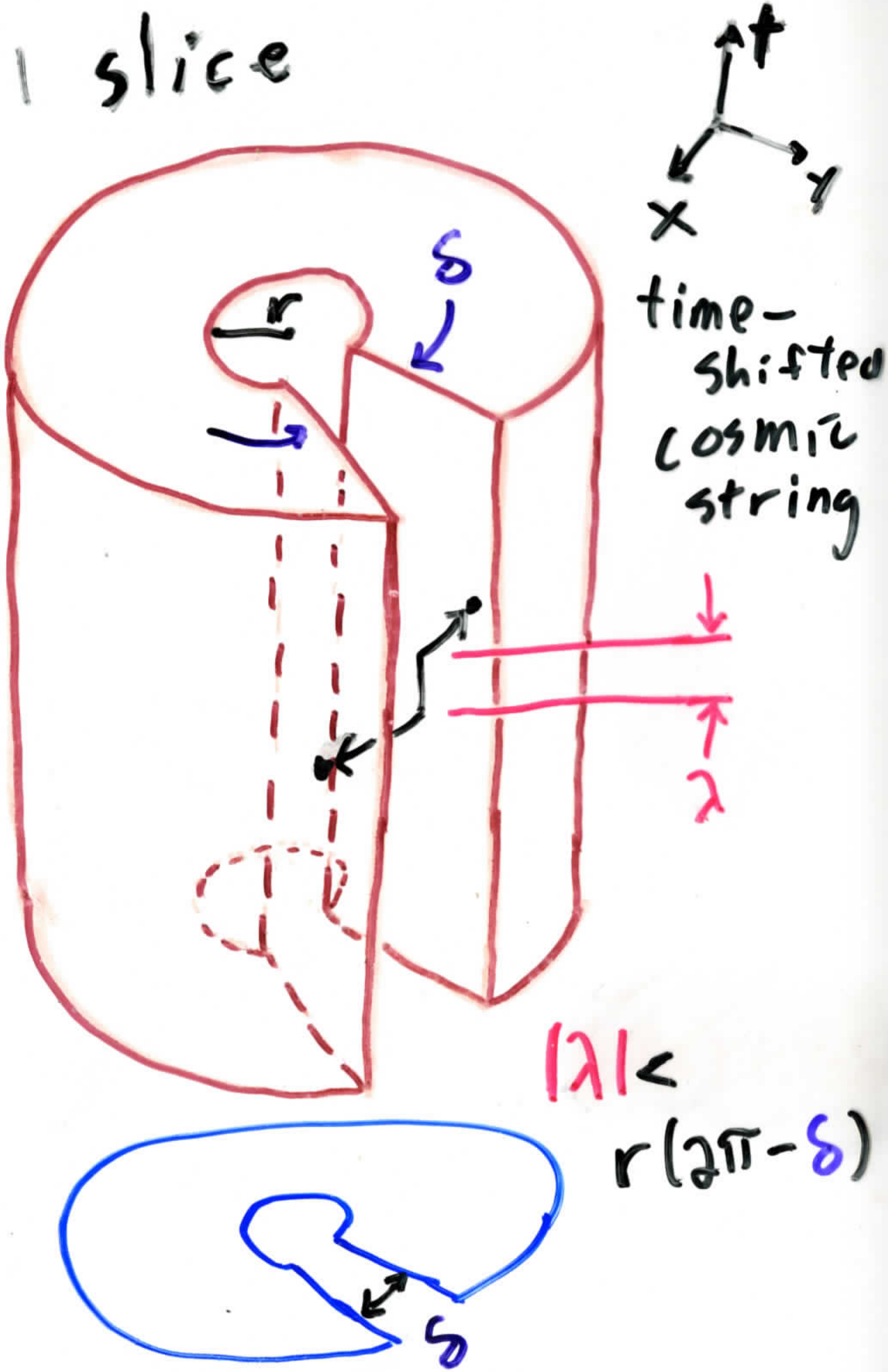
3+0



2+1 slice

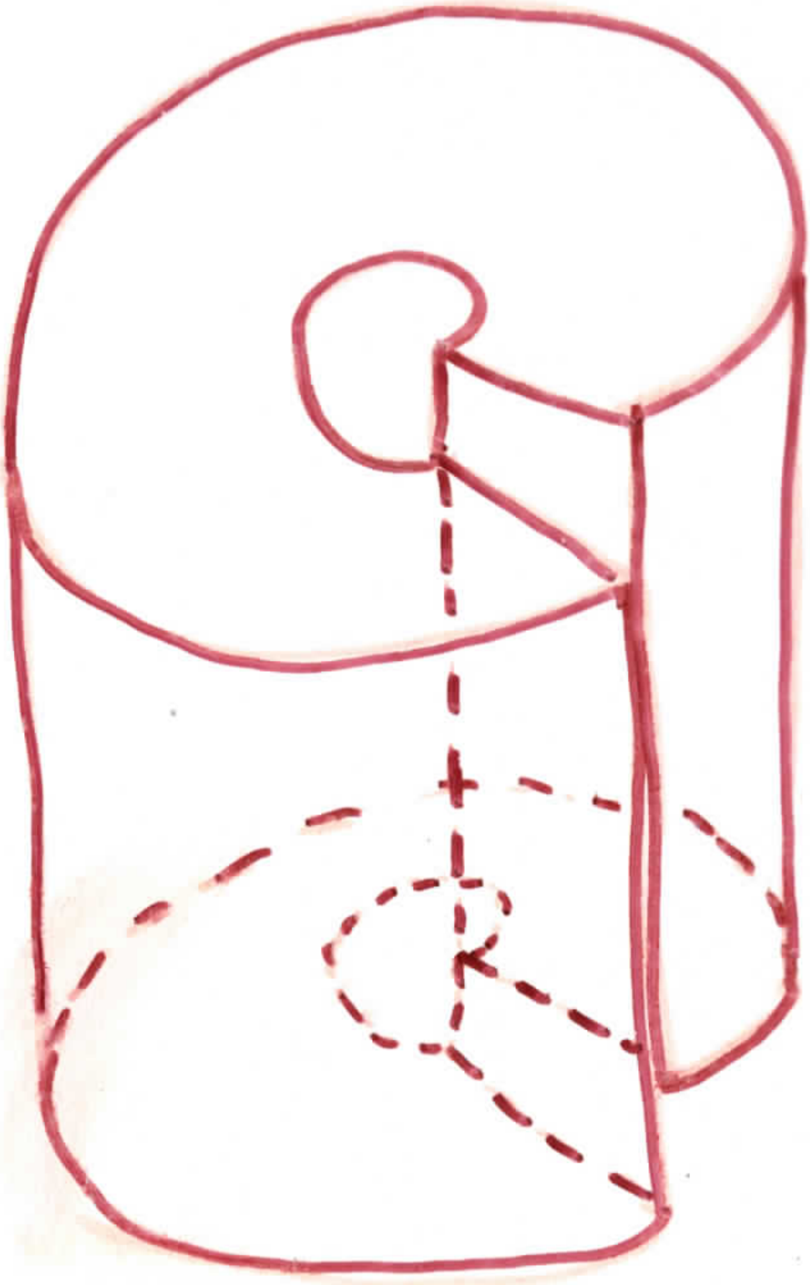


2+1 slice

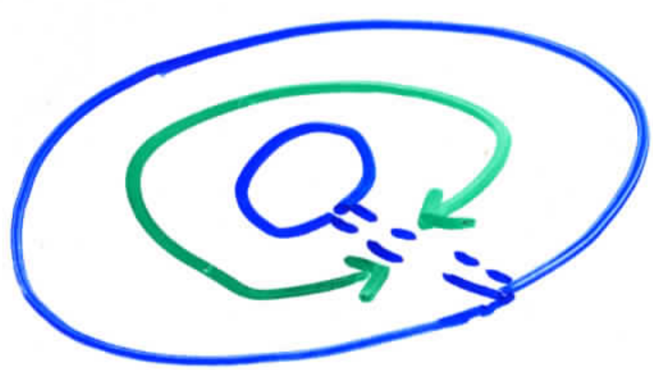


2+1

M



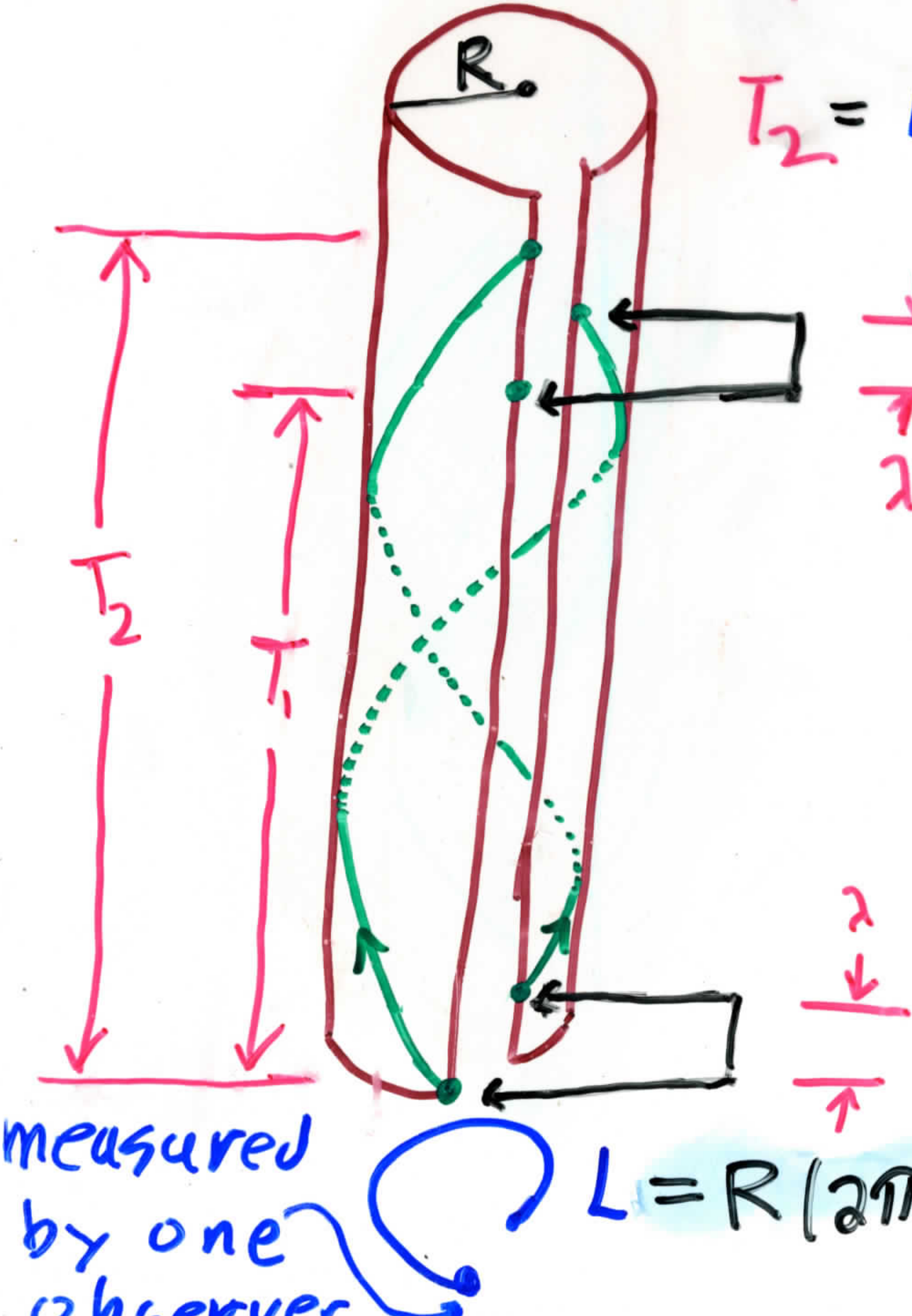
Q



$2\pi - \delta$

$T_1 = L - \lambda$

$T_2 = L + \lambda$



$L = R(2\pi - \delta)$

Causal curiosity:

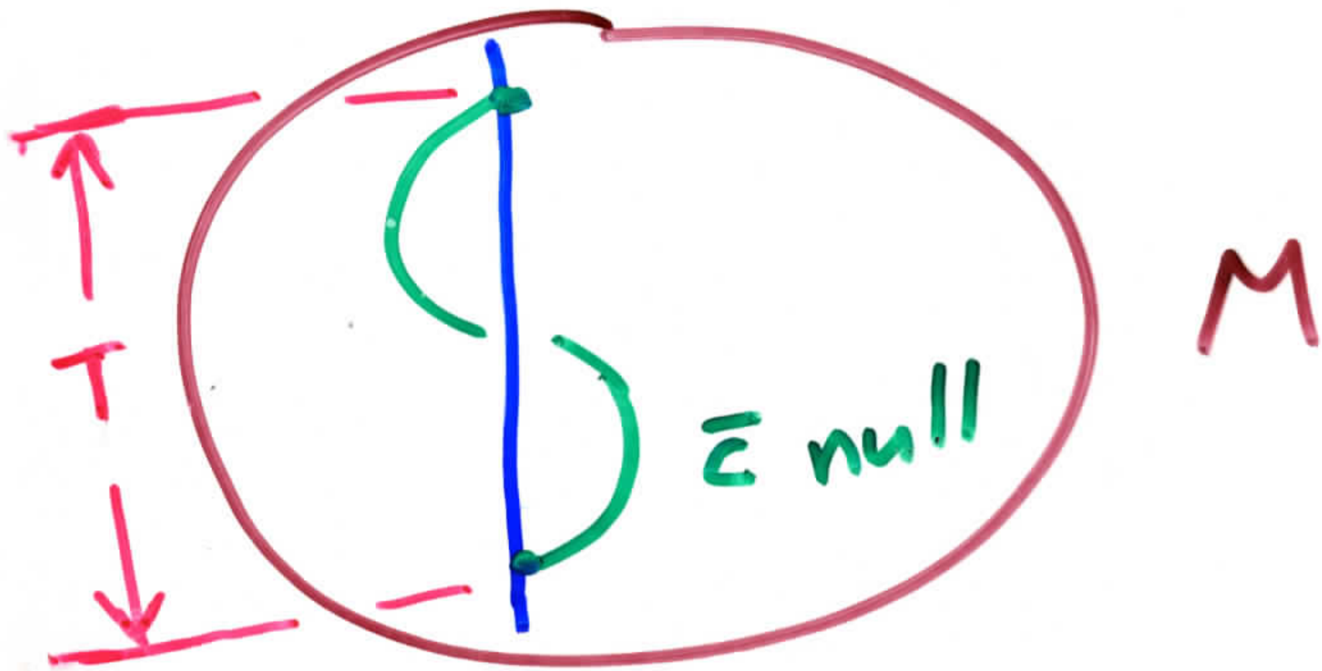
M ← GLOBALLY HYPERBOLIC

$\Big\downarrow$ quotient

M' assume: strongly causal

↑ GLOBALLY HYPERBOL.?

FUNDAMENTAL COCYCLE β

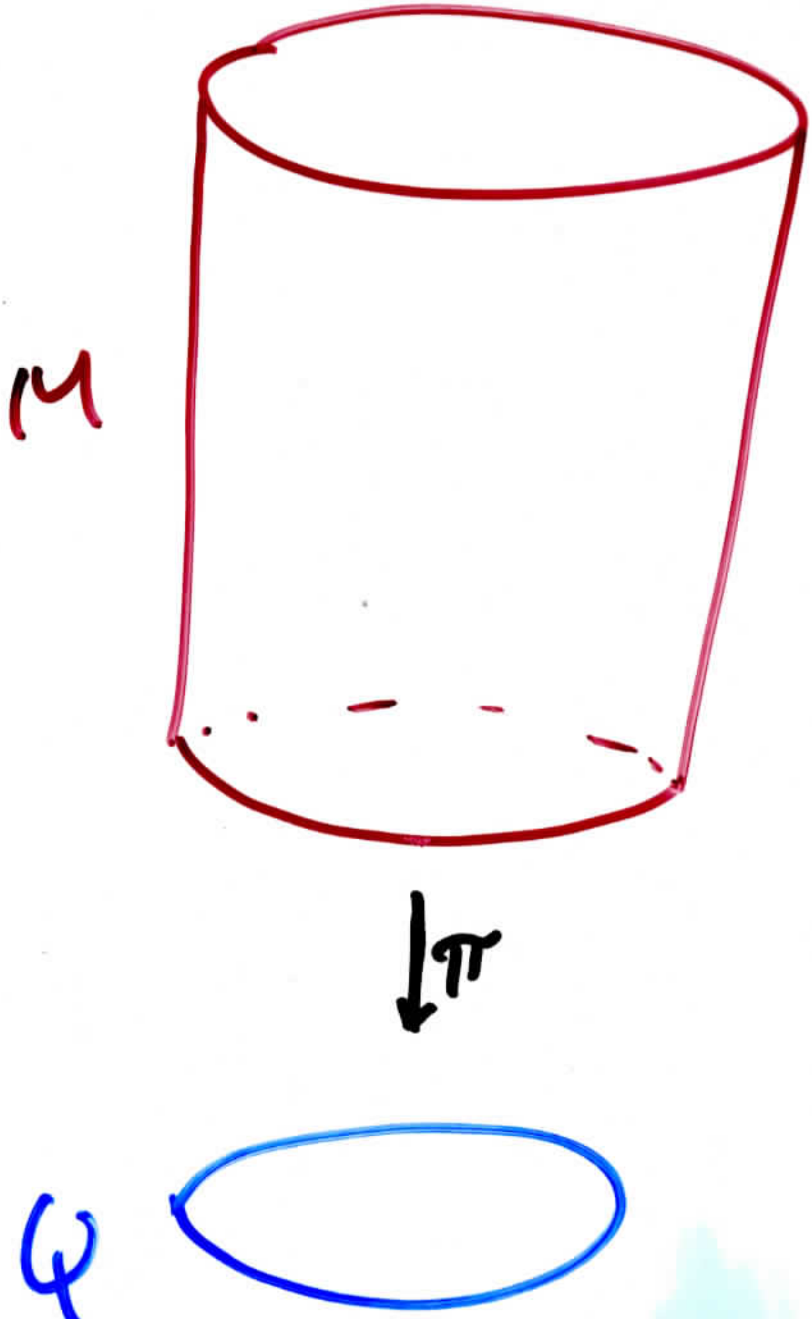


$\downarrow \pi$

$$\boxed{\beta(c) = T - L}$$

$L = L(c)$

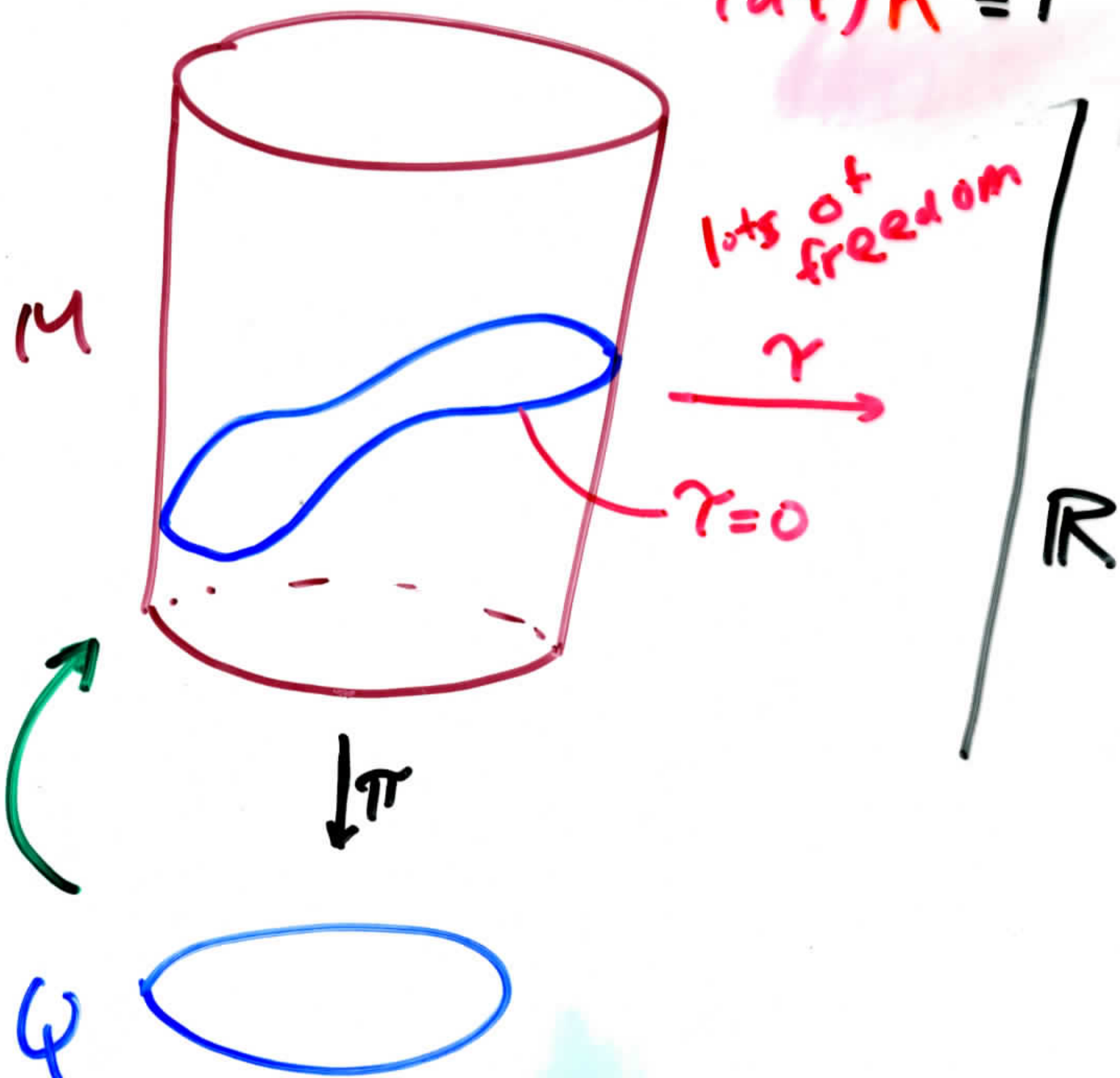
metric g on M



Riem. metric h_Q

metric g on M

$(d\tau)K = 1$



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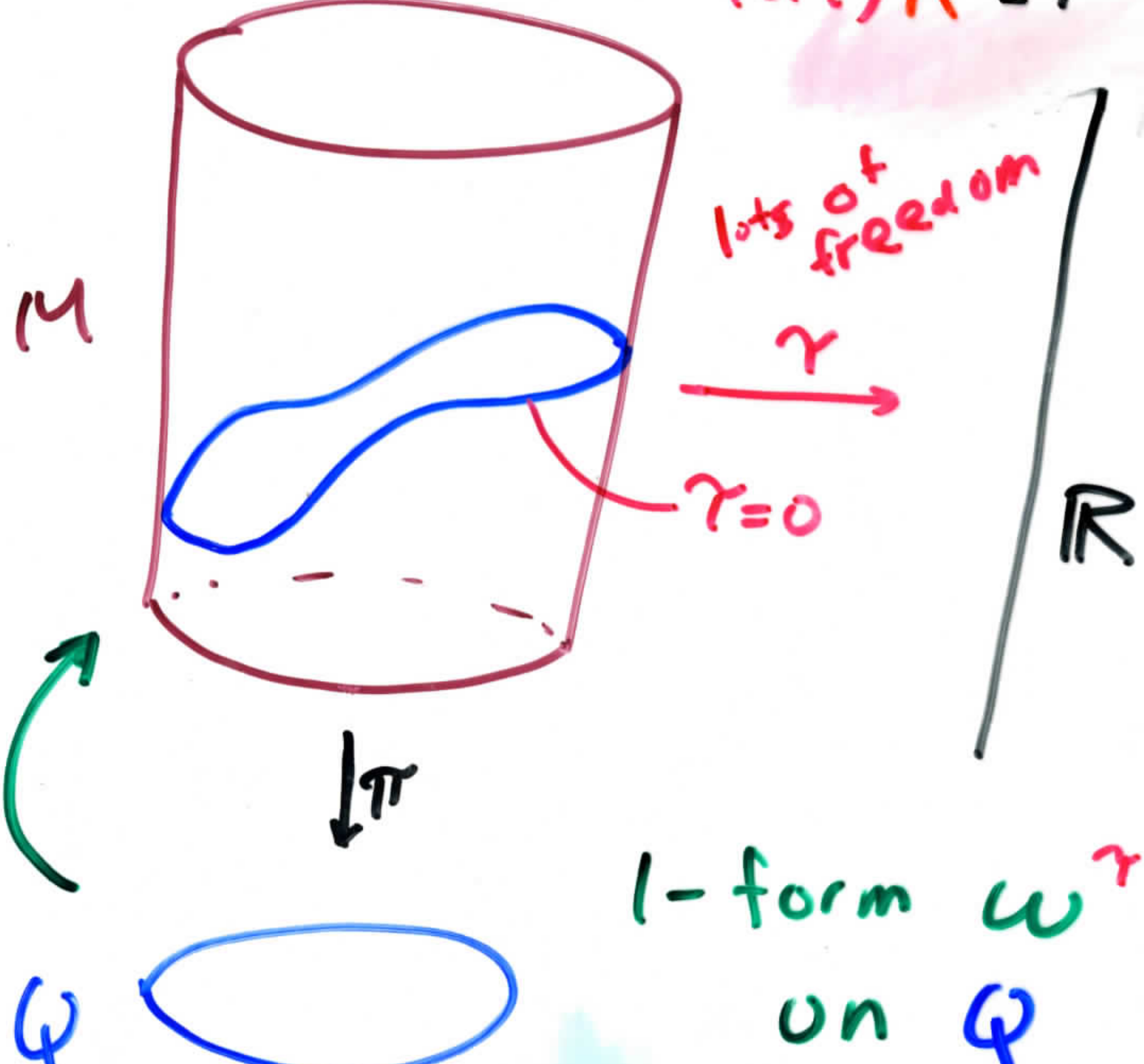
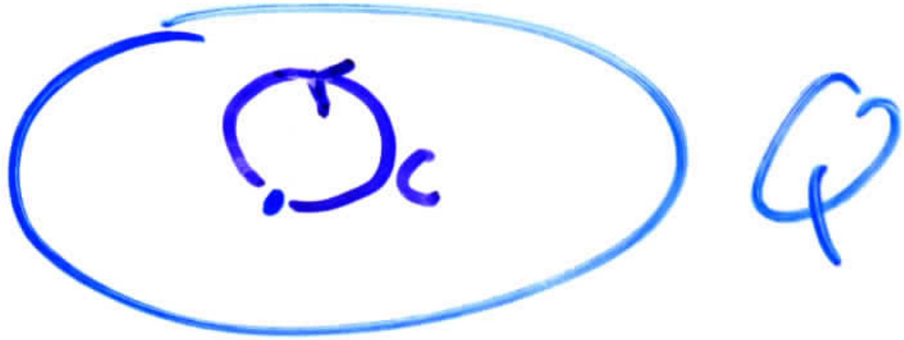


1-form ω^τ on Q (drift form)

Riem. metric h_Q

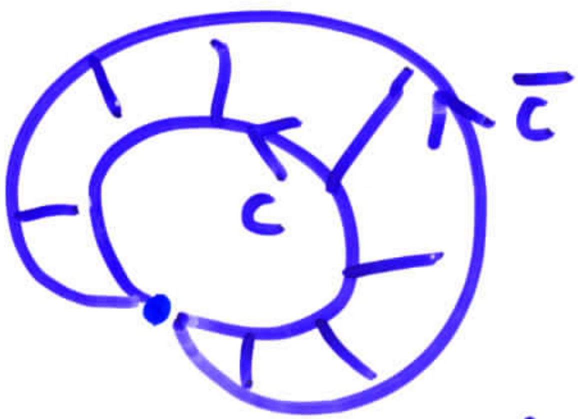
$$g = -|K|^2(d\tau + \omega)^2 + h_Q$$

static: $d\omega = 0$



$$\beta(c) = \int_c \omega$$

static: $(d\omega = 0)$

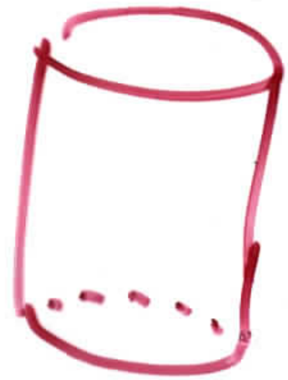


$c, \bar{c}$ homotopic $\Rightarrow$

$$\beta(c) = \beta(\bar{c})$$

$$\beta = [\omega] \in H^1_{dR}(Q)$$

$Q + \beta + |K|^2$



uniquely determine M

$\beta, \bar{\beta}$ agree on null-homotopic c

$\iff$ $M, \bar{M}$ share cover

$$\begin{tikzcd}
 & \tilde{M} \arrow[dl] \arrow[d] \\
M & \bar{M}
\end{tikzcd}$$

Weight of β

$$\mathrm{wt}(\beta) = \sup_c \frac{\beta(c)}{L(c)}$$

$\boxed{\text{RESULTS}}$

$\mathrm{wt}(\beta) > 1$: chrono. vicious

$\mathrm{wt}(\beta) < 1$: strongly causal

M globally hyerbolic $\Longleftrightarrow$ G complete

RESULTS

$wt(\beta) = 1$:

$\beta(c) = L(c)$: chron., not causal

$$\frac{\beta(c_n)}{L(c_n)} \to 1$$

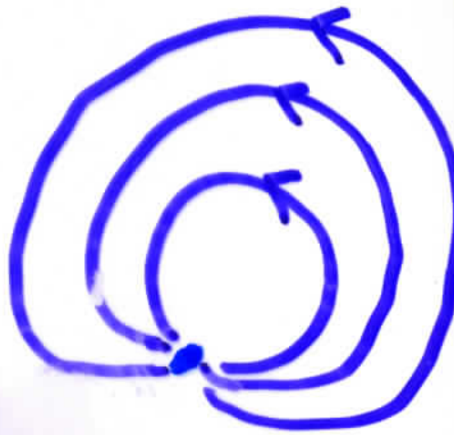
for some $\{c_n\}$

$L(c_n) - \beta(c_n) \to 0$:

causal, not strongly causal

RESULTS

$\mathrm{wt}(\beta) = 1$:



C_n

$$\frac{\beta(C_n)}{L(C_n)} \longrightarrow 1$$

for all $\{C_n\}$

$$L(C_n) - \beta(C_n) > \delta$$

for some $\{C_n\}$

$$L(C_n) - \beta(C_n) < A:$$

Strongl causal,
not glob. hyper.

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for all $\{C_n\}$

$$L(C_n) - \beta(C_n) \longrightarrow \infty:$$

glob. hyper. $\Longleftrightarrow$ φ complete

RESULTS

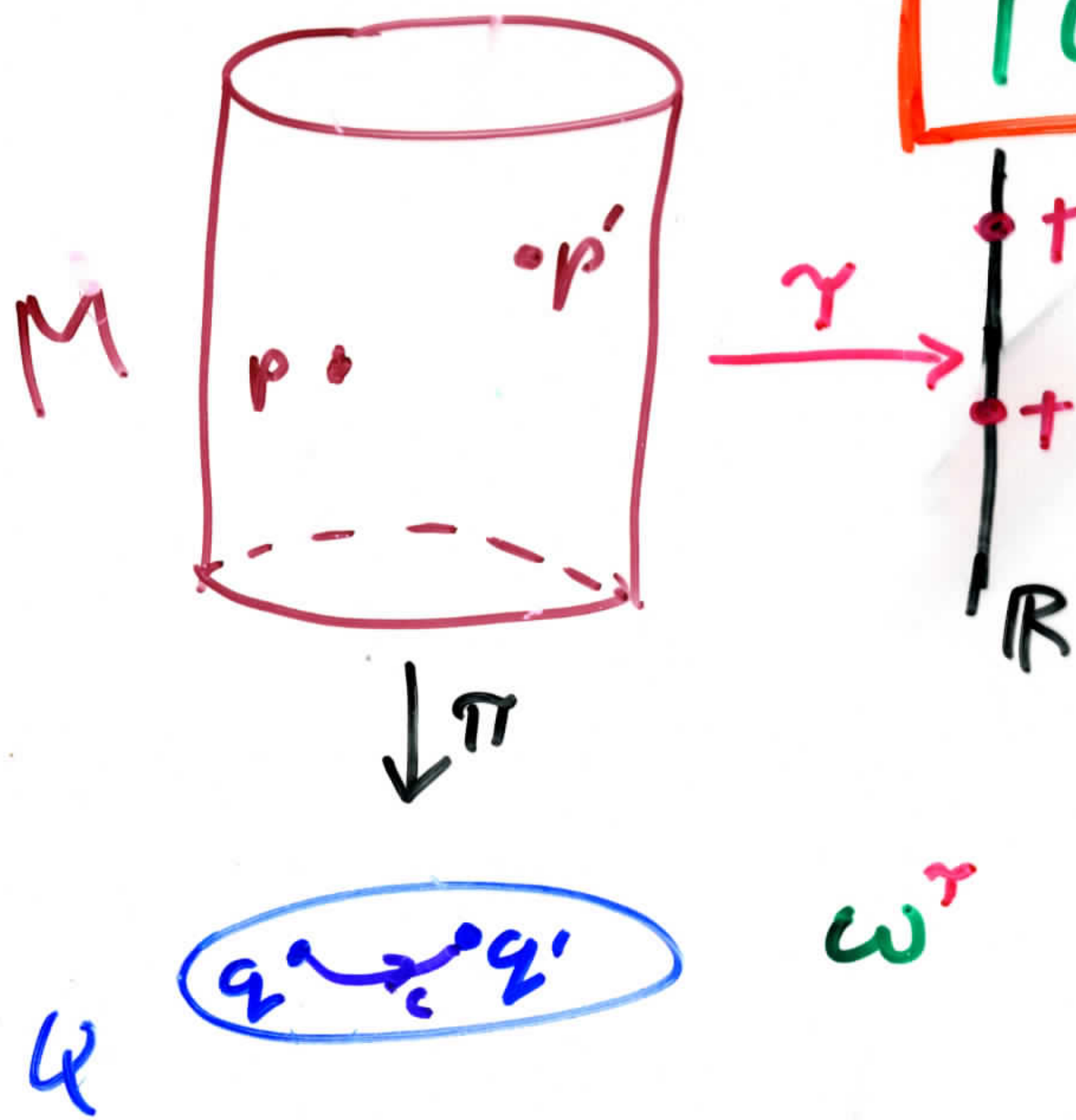
$$\begin{tikzcd} M \arrow[d, "\text{quotient}"] \\ M' \end{tikzcd} \qquad M \text{ glob. hyper.}$$

M, M' stationary

$\mathrm{wt}(\beta_{M'}) < 1 \implies$

M' glob. hyper.

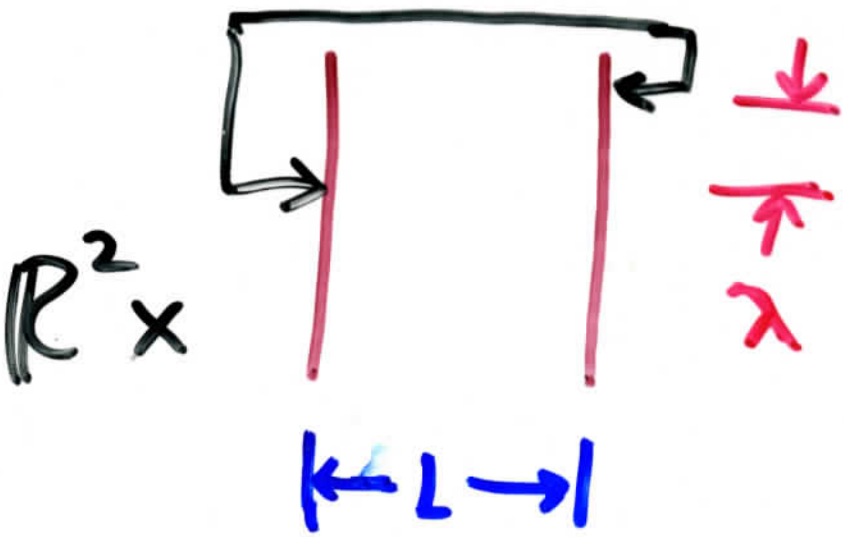
$\boxed{\text{TOOL}}$



$$p \ll p' \iff$$

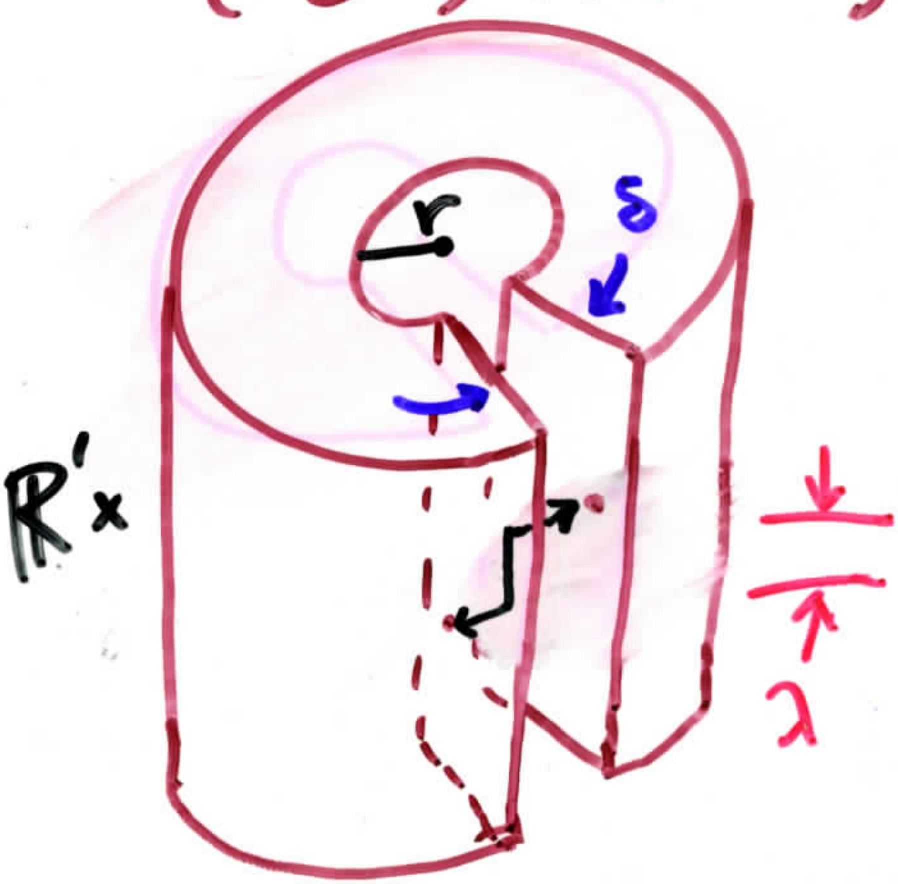
$$t' - t > \underbrace{\inf_{c} \left(L(c) - \int_c \omega \right)}_{\text{quasi-distance}}$$

3+1 Minkow. cylinder:



$$wt(\beta) = \frac{|\lambda|}{L}$$

cosmic string:



$$wt(\beta) = \frac{|\lambda|}{r(2\pi - \delta)}$$

Kerr $r > 2m$

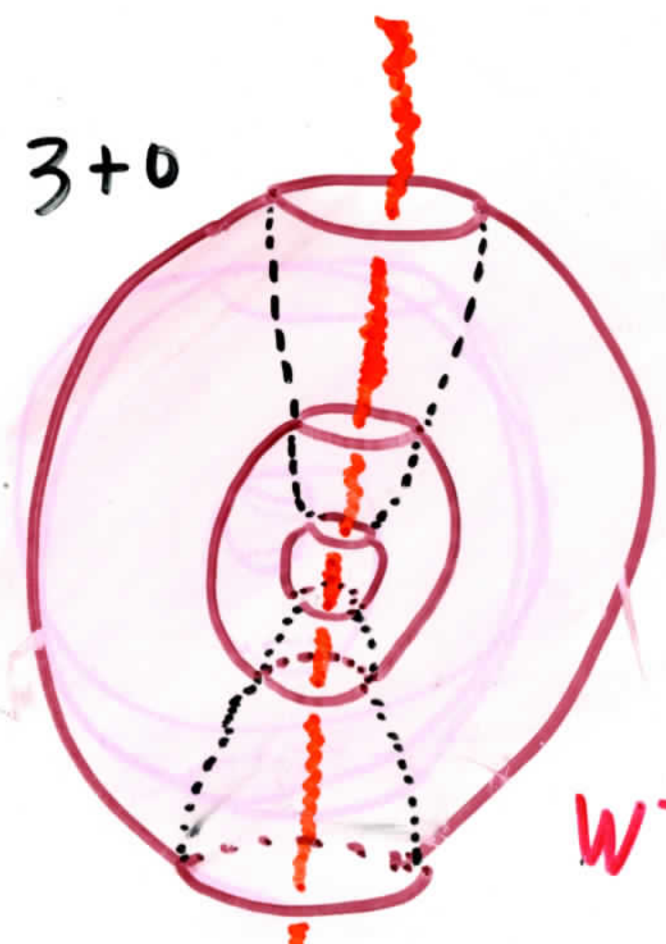
$a < m$

$\boxed{\text{Spinning black hole}}$

$wt(\mathcal{B}) = 1$

Kerr cosmic string

3+0



excise polar caps

angle $\to 0$ for $r \to \infty$

$wt(\mathcal{B}) =$ ad lib.

Kerr

$$\omega = \frac{2\,mra\,\sin^2\theta}{r^2 - 2mr + a^2\cos^2\theta}\,d\phi$$

ϕ = longitude

θ = co-latitude

arXiv:1412.7742