Recent Progress on Complex Quadric in Hermitian Symmetric Spaces

Young Jin Suh

Department of Mathematics
Kyungpook National University
Taegu 702-701, Korea

Departamento de Geometría y
Toplogía, Univesidad de Granada
18071 Granada, Spain
E-mail: yjsuh@knu.ac.kr

February 19, 2016

Contents

Contents

Hermitian Symmetric Spaces

HSSP: Hermitian Symmetric Space.

For HSSP of compact type with rank 1:
$\mathbf{C}P^m$, $\mathbf{Q}P^m$, $\mathbf{C}H^m$, and $\mathbf{Q}H^m$.

For HSSP of compact type with rank 2:
$SU(2+q)/S(U(2)\times U(q))$, $Q^m = G_2(\mathbf{R}^{2+p})$, $SO(8)/U(4)$, $Sp(2)/U(2)$ and $(\mathfrak{e}_{6(-78)}, \mathfrak{SO}(10)+\mathfrak{R})$ and of noncompact type $SU(2,q)/S(U(2)\times U(q))$, $Q^{m*} = G_2^*(\mathbf{R}^{2+p})$, $SO^*(8)/U(4)$, $Sp(2,\mathbf{R})/U(2)$ and $(\mathfrak{e}_{6(-14)}, \mathfrak{SO}(10)+\mathfrak{R})$ (See Helgason [6], [7]).

Hypersurfaces in Hermitian Symmetric Spaces

Let M be a hypersurfaces in a Hermitian Symmetric Space $\bar{M}$ with Kaehler structure J.
$SX = -\bar{\nabla}_X N$: Weingarten formula
Here we say S the shape operator of M in $\bar{M}$.

$\xi = -JN$: Reeb vector field.

$$JX = \phi X + \eta(X)N, \nabla_X \xi = \phi SX$$

for any vector field $X \in \Gamma(M)$.

Then (ϕ, ξ, η, g): almost contact structure on a hypersurface M

Define)

A hypersurfcae M: Isometric Reeb Flow $\Longleftrightarrow$ $\mathcal{L}_{\xi} g = 0$ $\Longleftrightarrow$ $g(d\phi_t X, d\phi_t Y) = g(X, Y)$ for any $X, Y \in \Gamma(M)$, where ϕ_t denotes a one parameter group, which is said to be an isometric Reeb flow of M, defined by

$$\frac{d\phi_t}{dt} = \xi(\phi_t(p)), \quad \phi_0(p) = p, \dot{\phi}_0(p) = \xi(p).$$

Note)

$\mathcal{L}_{\xi} g = 0$ $\Longleftrightarrow$ $\nabla_j \xi_i + \nabla_i \xi_j = 0$, $\nabla \xi$: skew-symmetric $\Longleftrightarrow$ $g(\nabla_X \xi, Y) + g(\nabla_Y \xi, X) = 0$ $\Longleftrightarrow$ $g((\phi S - S\phi)X, Y) = 0$ for any $X, Y \in \Gamma(M)$.

Define)

A hypersurfcae M: Isometric Reeb Flow $\iff$ $\mathcal{L}_\xi g = 0$ $\iff$ $g(d\phi_t X, d\phi_t Y) = g(X, Y)$ for any $X, Y \in \Gamma(M)$, where ϕ_t denotes a one parameter group, which is said to be an isometric Reeb flow of M, defined by

$$\frac{d\phi_t}{dt} = \xi(\phi_t(p)), \quad \phi_0(p) = p, \dot{\phi}_0(p) = \xi(p).$$

Note)

$\mathcal{L}_\xi g = 0 \iff \nabla_j \xi_i + \nabla_i \xi_j = 0$, $\nabla\xi$: skew-symmetric $\iff$ $g(\nabla_X \xi, Y) + g(\nabla_Y \xi, X) = 0 \iff g((\phi S - S\phi)X, Y) = 0$ for any $X, Y \in \Gamma(M)$.

Define)

A hypersurfcae M in Kaehler manifold $\bar{M}$ is contact $\iff$ there exists a nonvanishing smooth function ρ on M such that $d\eta = 2\rho\omega$. Then it is clear $\eta \wedge (d\eta)^{m-1} \neq 0$.

Note)

The equation $d\eta = 2\rho\omega$ means that $d\eta(X, Y) = 2\rho g(\phi X, Y)$ for any vector fields X, Y on M.

Note)

$d\eta(X, Y) = d(\eta(Y))(X) - d(\eta(X))(Y) - \eta([X, Y]) \iff g((S\phi + \phi S)X, Y) = 2\rho g(\phi X, Y)$.

Define)

A hypersurfcae M in Kaehler manifold $\bar{M}$ is contact $\Longleftrightarrow$ there exists a nonvanishing smooth function ρ on M such that $d\eta = 2\rho\omega$. Then it is clear $\eta \wedge (d\eta)^{m-1} \neq 0$.

Note)

The equation $d\eta = 2\rho\omega$ means that $d\eta(X, Y) = 2\rho g(\phi X, Y)$ for any vector fields X, Y on M.

Note)

$d\eta(X, Y) = d(\eta(Y))(X) - d(\eta(X))(Y) - \eta([X, Y]) \Longleftrightarrow g((S\phi + \phi S)X, Y) = 2\rho g(\phi X, Y)$.

Define)

A hypersurfcae M in Kaehler manifold $\bar{M}$ is contact $\iff$ there exists a nonvanishing smooth function ρ on M such that $d\eta = 2\rho\omega$. Then it is clear $\eta\wedge(d\eta)^{m-1}\neq 0$.

Note)

The equation $d\eta = 2\rho\omega$ means that $d\eta(X, Y) = 2\rho g(\phi X, Y)$ for any vector fields X, Y on M.

Note)

$d\eta(X, Y) = d(\eta(Y))(X) - d(\eta(X))(Y) - \eta([X, Y]) \iff g((S\phi + \phi S)X, Y) = 2\rho g(\phi X, Y)$.

Problem 1

Classify all of homogeneous hypersurfaces in Hermitian Symmetric Spaces.

In this talk, we consider the following problems:

Problem 2

If M is a connected hypersurface in Hermitian symmetric spaces $\bar{M}$ with isometric Reeb flow, then M becomes homogeneous ?

Problem 3

If M is a connected contact hypersurface in Hermitian symmetric spaces $\bar{M}$, then M becomes homogeneous ?

Problem 1

Classify all of homogeneous hypersurfaces in Hermitian Symmetric Spaces.

In this talk, we consider the following problems:

Problem 2

If M is a connected hypersurface in Hermitian symmetric spaces $\bar{M}$ with isometric Reeb flow, then M becomes homogeneous ?

Problem 3

If M is a connected contact hypersurface in Hermitian symmetric spaces $\bar{M}$, then M becomes homogeneous ?

Problem 1

Classify all of homogeneous hypersurfaces in Hermitian Symmetric Spaces.

In this talk, we consider the following problems:

Problem 2

If M is a connected hypersurface in Hermitian symmetric spaces $\bar{M}$ with isometric Reeb flow, then M becomes homogeneous ?

Problem 3

If M is a connected contact hypersurface in Hermitian symmetric spaces $\bar{M}$, then M becomes homogeneous ?

Isometric Reeb Flow

- Note 1) For hypersurfaces in $\mathbf{C}P^m$, $\mathbf{C}H^m$ and $\mathbf{Q}P^m$ with isometric Reeb flow:
  Okumura 1976, Montiel and Romero 1986, Perez and Martinez 1986 respectively.
- Note 2) For hypersurfaces in $G_2(\mathbf{C}^{m+2})$, $G_2^*(\mathbf{C}^{m+2})$ and $Q^m = SO(m+2)/SO(2)SO(m)$ with isometric Reeb flow:
  Berndt and Suh, 2002 and 2012, Suh, 2013, Berndt and Suh, 2013.

Isometric Reeb Flow

- Note 1) For hypersurfaces in $\mathbf{C}P^m$, $\mathbf{C}H^m$ and $\mathbf{Q}P^m$ with isometric Reeb flow:
  Okumura 1976, Montiel and Romero 1986, Perez and Martinez 1986 respectively.
- Note 2) For hypersurfaces in $G_2(\mathbf{C}^{m+2})$, $G_2^*(\mathbf{C}^{m+2})$ and $Q^m = SO(m+2)/SO(2)SO(m)$ with isometric Reeb flow:
  Berndt and Suh, 2002 and 2012, Suh, 2013, Berndt and Suh, 2013.

Contact Hypersurfaces

- Note 1) For contact hypersurfaces in $\mathbf{C}^m$, $\mathbf{C}P^m$ and $\mathbf{C}H^m$: Okumura 1966, and Vernon 1987.
- Note 2) Recently, a contact hypersurface in $G_2(\mathbf{C}^{m+2})$, $G_2^*(\mathbf{C}^{m+2})$, complex quadric $Q^m = SO(m+2)/SO(2)SO(m)$ and noncompact complex quadric $Q^{m*} = SO^0(2,m)/SO(2)SO(m)$: Suh, 2006, 2014, 2015, 2016 and Berndt, Lee and Suh, 2013, Berndt and Suh, 2014, 2015.

Contact Hypersurfaces

- Note 1) For contact hypersurfaces in $\mathbf{C}^m$, $\mathbf{C}P^m$ and $\mathbf{C}H^m$: Okumura 1966, and Vernon 1987.
- Note 2) Recently, a contact hypersurface in $G_2(\mathbf{C}^{m+2})$, $G_2^*(\mathbf{C}^{m+2})$, complex quadric $Q^m = SO(m+2)/SO(2)SO(m)$ and noncompact complex quadric $Q^{m*} = SO^0(2,m)/SO(2)SO(m)$: Suh, 2006, 2014, 2015, 2016 and Berndt, Lee and Suh, 2013, Berndt and Suh, 2014, 2015.

Three directions of our talk

In this talk, we introduce some recent results for hypersurface M in $Q^m = SO(m+2)/SO(2){\cdot}SO(m)$ and $Q^{*m} = SO^0(2,m)/SO(2){\cdot}SO(m)$ as follows:

- **1)** A classification problem for hypersurfaces with isometric Reeb flow in *complex quadric* Q^m.
- **2)** A classification problem for contact hypersurfaces in *non-compact complex quadric* Q^{*m}.
- **3)** A classification problem for hypersurfaces with harmonic curvature in *complex quadric* Q^m.
- **4)** Pseudo-Einstein, Pseudo-anti commuting and Ricci soliton problems in *complex quadric* Q^m.

Three directions of our talk

In this talk, we introduce some recent results for hypersurface M in $Q^m = SO(m+2)/SO(2)\cdot SO(m)$ and ${Q^*}^m = SO^0(2,m)/SO(2)\cdot SO(m)$ as follows:

- **1)** A classification problem for hypersurfaces with isometric Reeb flow in *complex quadric* Q^m.
- **2)** A classification problem for contact hypersurfaces in *non-compact complex quadric* ${Q^*}^m$.
- **3)** A classification problem for hypersurfaces with harmonic curvature in *complex quadric* Q^m.
- **4)** Pseudo-Einstein, Pseudo-anti commuting and Ricci soliton problems in *complex quadric* Q^m.

Three directions of our talk

In this talk, we introduce some recent results for hypersurface M in $Q^m = SO(m+2)/SO(2){\cdot}SO(m)$ and $Q^{*m} = SO^0(2,m)/SO(2){\cdot}SO(m)$ as follows:

- **1)** A classification problem for hypersurfaces with isometric Reeb flow in *complex quadric* Q^m.
- **2)** A classification problem for contact hypersurfaces in *non-compact complex quadric* Q^{*m}.
- **3)** A classification problem for hypersurfaces with harmonic curvature in *complex quadric* Q^m.
- **4)** Pseudo-Einstein, Pseudo-anti commuting and Ricci soliton problems in *complex quadric* Q^m.

Three directions of our talk

In this talk, we introduce some recent results for hypersurface M in $Q^m = SO(m+2)/SO(2) \cdot SO(m)$ and ${Q^*}^m = SO^0(2,m)/SO(2) \cdot SO(m)$ as follows:

- **1)** A classification problem for hypersurfaces with isometric Reeb flow in *complex quadric* Q^m.
- **2)** A classification problem for contact hypersurfaces in *non-compact complex quadric* ${Q^*}^m$.
- **3)** A classification problem for hypersurfaces with harmonic curvature in *complex quadric* Q^m.
- **4)** Pseudo-Einstein, Pseudo-anti commuting and Ricci soliton problems in *complex quadric* Q^m.

The homogeneous quadratic equation

$$Q^m = \{z \in \mathbb{C}^{m+2} | z_1^2 + \ldots + z_{m+2}^2 = 0\} \subset \mathbb{C}P^{m+1}$$

defines a complex hypersurface in complex projective space $\mathbb{C}P^{m+1} = SU_{m+2}/S(U_{m+1}U_1)$.

For a unit normal vector N of Q^m at a point $[z] \in Q^m$ we denote by A_N the shape operator of Q^m in $\mathbb{C}P^{m+1}$ with respect to N.

The shape operator is an involution on $T_{[z]}Q^m$ and $T_{[z]}Q^m = V(A_N) \oplus JV(A_N)$, where $V(A_N)$ is the $(+1)$-eigenspace and $JV(A_N)$ is the (-1)-eigenspace of A_N.

Geometrically this means that A_N defines a real structure on the complex vector space $T_{[z]}Q^m$, or equivalently, is a complex conjugation on $T_{[z]}Q^m$.

The Riemannian curvature tensor R of Q^m can be expressed as follows:

$$\begin{aligned} R(X,Y)Z &= g(Y,Z)X - g(X,Z)Y + g(JY,Z)JX \\ &\quad - g(JX,Z)JY - 2g(JX,Y)JZ \\ &\quad + g(AY,Z)AX - g(AX,Z)AY \\ &\quad + g(JAY,Z)JAX - g(JAX,Z)JAY. \end{aligned}$$

A nonzero tangent vector $W \in T_{[z]}Q^m$ is called singular if it is tangent to more than one maximal flat in Q^m.

1. If a conjugation $A \in \mathfrak{A}_{[z]}$ such that $W \in V(A)$, then W is singular, that is $\mathfrak{A}$-principal.
2. If a conjugation $A \in \mathfrak{A}_{[z]}$ and orthonormal vectors $X, Y \in V(A)$ such that $W/||W|| = (X + JY)/\sqrt{2}$, then W is said to be $\mathfrak{A}$-isotropic.

The Riemannian curvature tensor R of Q^m can be expressed as follows:

$$\begin{aligned} R(X,Y)Z &= g(Y,Z)X - g(X,Z)Y + g(JY,Z)JX \\ &\quad -g(JX,Z)JY - 2g(JX,Y)JZ \\ &\quad +g(AY,Z)AX - g(AX,Z)AY \\ &\quad +g(JAY,Z)JAX - g(JAX,Z)JAY. \end{aligned}$$

A nonzero tangent vector $W \in T_{[z]}Q^m$ is called singular if it is tangent to more than one maximal flat in Q^m.

1. If a conjugation $A \in \mathfrak{A}_{[z]}$ such that $W \in V(A)$, then W is singular, that is $\mathfrak{A}$-principal.
2. If a conjugation $A \in \mathfrak{A}_{[z]}$ and orthonormal vectors $X, Y \in V(A)$ such that $W/||W|| = (X + JY)/\sqrt{2}$, then W is said to be $\mathfrak{A}$-isotropic.

Geometric Descriptions of the Tube

We assume that m is even, say $m = 2k$. The map

$$\mathbb{C}P^k \to Q^{2k} \subset \mathbb{C}P^{2k+1}\ ,\ [z_1, \dots, z_{k+1}] \mapsto [z_1, \dots, z_{k+1}, iz_1, \dots, iz_{k+1}]$$

gives an embedding of $\mathbb{C}P^k$ into Q^{2k} as a totally geodesic complex submanifold.

Define a complex structure j on $\mathbb{C}^{2k+2}$ by

$$j(z_1, \dots, z_{k+1}, z_{k+2}, \dots, z_{2k+2}) = (-z_{k+2}, \dots, -z_{2k+2}, z_1, \dots, z_{k+1}).$$

Then $j^2 = -I$ and note that $ij = ji$. We can then identify $\mathbb{C}^{2k+2}$ with $\mathbb{C}^{k+1} \oplus j\mathbb{C}^{k+1}$ and get

$$T_{[z]}\mathbb{C}P^k = \{X + ijX | X \in V(A_{\bar{z}})\} \quad (\mathfrak{A} - isotropic).$$

The normal space becomes ($\mathfrak{A}$ − *isotropic*) as follows:

$$\nu_{[z]}\mathbb{C}P^k = A_{\bar{z}}(T_{[z]}\mathbf{C}P^k) = \{X - ijX | X \in V(A_{\bar{z}})\}.$$

Since N is $\mathfrak{A}$-isotropic, the four vectors $\{N, JN, AN, JAN\}$ become pairwise orthonormal and the normal Jacobi operator R_N is given by

$$\begin{aligned} R_N Z &= R(Z, N)N \\ &= Z - g(Z, N)N + 3g(Z, JN)JN \\ &\quad -g(Z, AN)AN - g(Z, JAN)JAN. \end{aligned}$$

Both $T_{[z]}\mathbb{C}P^k$ and $\nu_{[z]}\mathbb{C}P^k$ are invariant under R_N, and R_N has three eigenvalues $0, 1, 4$ according to $\mathbf{R}N \oplus [AN]$, $T_{[z]}Q^{2k} \ominus ([N] \oplus [AN])$ and $\mathbf{R}JN$.

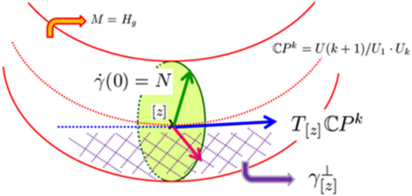
Normal geodesic in complex quadrics

M : an open part of a tube over $\mathbb{C}P^k$ in Q^{2k}

Let γ be a geodesic in Q^{2k} with

$\gamma(0) = [z]$ and $\dot\gamma(0) = N$.

Denote $\gamma^{\perp}_{[z]} = T_{[z]}\mathbb{C}P^k \oplus (\nu_{[z]}\mathbb{C}P^k \ominus \mathbb{R}N)$



Principal Curvatures and Spaces of the Tube

To calculate the principal curvatures of the tube of radius $0 < r < \pi/2$ around $\mathbb{C}P^k$ we use the standard Jacobi field method as described in Section 8.2 of Berndt, Console and Olmos.

Let γ be the geodesic in Q^{2k} with $\gamma(0) = [z]$ and $\dot\gamma(0) = N$ and denote by $\gamma^\perp$ the parallel subbundle of TQ^{2k} along γ defined by $\gamma^\perp_{\gamma(t)} = T_{[\gamma(t)]}Q^{2k} \ominus \mathbb{R}\dot\gamma(t)$. Moreover, define the $\gamma^\perp$-valued tensor field $R^\perp_\gamma$ along γ by $R^\perp_{\gamma(t)}X = R(X, \dot\gamma(t))\dot\gamma(t)$. Now consider the $\mathrm{End}(\gamma^\perp)$-valued differential equation

$$Y'' + R^\perp_\gamma \circ Y = 0.$$

Let D be the unique solution of this differential equation with initial values

$$D(0) = \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix}, \ D'(0) = \begin{pmatrix} 0 & 0 \\ 0 & I \end{pmatrix},$$

where the decomposition of the matrices is with respect to

$$\gamma_{[z]}^{\perp} = T_{[z]}(M) = T_{[z]}\mathbb{C}P^k \oplus (\nu_{[z]}\mathbb{C}P^k \ominus \mathbb{R}N)$$

and I denotes the identity transformation on the corresponding space. Then the shape operator $S(r)$ of the tube of radius $0 < r < \pi/2$ around $\mathbb{C}P^k$ with respect to $\dot{\gamma}(r)$ is given by

$$S(r) = -D'(r) \circ D^{-1}(r).$$

Let D be the unique solution of this differential equation with initial values

$$D(0) = \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix},\ D'(0) = \begin{pmatrix} 0 & 0 \\ 0 & I \end{pmatrix},$$

where the decomposition of the matrices is with respect to

$$\gamma_{[z]}^{\perp} = T_{[z]}(M) = T_{[z]}\mathbb{C}P^k \oplus (\nu_{[z]}\mathbb{C}P^k \ominus \mathbb{R}N)$$

and I denotes the identity transformation on the corresponding space. Then the shape operator $S(r)$ of the tube of radius $0 < r < \pi/2$ around $\mathbb{C}P^k$ with respect to $\dot\gamma(r)$ is given by

$$S(r) = -D'(r) \circ D^{-1}(r).$$

Let D be the unique solution of this differential equation with initial values

$$D(0) = \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix}, \; D'(0) = \begin{pmatrix} 0 & 0 \\ 0 & I \end{pmatrix},$$

where the decomposition of the matrices is with respect to

$$\gamma_{[z]}^{\perp} = T_{[z]}(M) = T_{[z]}\mathbb{C}P^k \oplus (\nu_{[z]}\mathbb{C}P^k \ominus \mathbb{R}N)$$

and I denotes the identity transformation on the corresponding space. Then the shape operator $S(r)$ of the tube of radius $0 < r < \pi/2$ around $\mathbb{C}P^k$ with respect to $\dot{\gamma}(r)$ is given by

$$S(r) = -D'(r) \circ D^{-1}(r).$$

If we decompose $\gamma_{[z]}^{\perp}$ further into

$$\gamma_{[z]}^{\perp} = [AN] \oplus (T_{[z]}\mathbb{C}P^k \ominus [AN]) \oplus (\nu_{[z]}\mathbb{C}P^k \ominus [N]) \oplus \mathbb{R}JN,$$

we get by explicit computation that

$$S(r) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \tan(r) & 0 & 0 \\ 0 & 0 & -\cot(r) & 0 \\ 0 & 0 & 0 & -2\cot(2r) \end{pmatrix}$$

with respect to that decomposition $T_{[z]}M = \gamma_{[z]}^{\perp}$, $[z]\in M$.

If we decompose $\gamma_{[z]}^{\perp}$ further into

$$\gamma_{[z]}^{\perp} = [AN] \oplus (T_{[z]}\mathbb{C}P^k \ominus [AN]) \oplus (\nu_{[z]}\mathbb{C}P^k \ominus [N]) \oplus \mathbb{R}JN,$$

we get by explicit computation that

$$S(r) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \tan(r) & 0 & 0 \\ 0 & 0 & -\cot(r) & 0 \\ 0 & 0 & 0 & -2\cot(2r) \end{pmatrix}$$

with respect to that decomposition $T_{[z]}M = \gamma_{[z]}^{\perp}$, $[z] \in M$.

Proposition 3.1. (Berndt and Suh, IJM., 2013)

Let M be the tube of radius $0 < r < \pi/2$ around the totally geodesic $\mathbb{C}P^k$ in Q^{2k}. Then the following hold:

1. M is a Hopf hypersurface.
2. The normal bundle of M consists of $\mathfrak{A}$-isotropic singular.
3. M has four distinct constant principal curvatures.

principal curvature	eigenspace	multiplicity
0	$\mathcal{C} \ominus \mathcal{Q}$	2
$\tan(r)$	$T\mathbb{C}P^k \ominus (\mathcal{C} \ominus \mathcal{Q})$	$2k-2$
$-\cot(r)$	$\nu\mathbb{C}P^k \ominus \mathbb{C}\nu M$	$2k-2$
$-2\cot(2r)$	$\mathcal{F}$	1

4. $S\phi = \phi S$.
5. The Reeb flow on M is an isometric flow.

Proposition 3.1. (Berndt and Suh, IJM., 2013)

Let M be the tube of radius $0 < r < \pi/2$ around the totally geodesic $\mathbb{C}P^k$ in Q^{2k}. Then the following hold:

1. M is a Hopf hypersurface.
2. The normal bundle of M consists of $\mathfrak{A}$-isotropic singular.
3. M has four distinct constant principal curvatures.

principal curvature	eigenspace	multiplicity
0	$\mathcal{C} \ominus \mathcal{Q}$	2
$\tan(r)$	$T\mathbb{C}P^k \ominus (\mathcal{C} \ominus \mathcal{Q})$	$2k-2$
$-\cot(r)$	$\nu\mathbb{C}P^k \ominus \mathbb{C}\nu M$	$2k-2$
$-2\cot(2r)$	$\mathcal{F}$	1

4. $S\phi = \phi S$.
5. The Reeb flow on M is an isometric flow.

Proposition 3.1. (Berndt and Suh, IJM., 2013)

Let M be the tube of radius $0 < r < \pi/2$ around the totally geodesic $\mathbb{C}P^k$ in Q^{2k}. Then the following hold:

1. M is a Hopf hypersurface.
2. The normal bundle of M consists of $\mathfrak{A}$-isotropic singular.
3. M has four distinct constant principal curvatures.

principal curvature	eigenspace	multiplicity
0	$\mathcal{C} \ominus \mathcal{Q}$	2
$\tan(r)$	$T\mathbb{C}P^k \ominus (\mathcal{C} \ominus \mathcal{Q})$	$2k-2$
$-\cot(r)$	$\nu\mathbb{C}P^k \ominus \mathbb{C}\nu M$	$2k-2$
$-2\cot(2r)$	$\mathcal{F}$	1

4. $S\phi = \phi S$.
5. The Reeb flow on M is an isometric flow.

Proposition 3.1. (Berndt and Suh, IJM., 2013)

Let M be the tube of radius $0 < r < \pi/2$ around the totally geodesic $\mathbb{C}P^k$ in Q^{2k}. Then the following hold:

1. M is a Hopf hypersurface.
2. The normal bundle of M consists of $\mathfrak{A}$-isotropic singular.
3. M has four distinct constant principal curvatures.

principal curvature	eigenspace	multiplicity
0	$\mathcal{C} \ominus \mathcal{Q}$	2
$\tan(r)$	$T\mathbb{C}P^k \ominus (\mathcal{C} \ominus \mathcal{Q})$	$2k-2$
$-\cot(r)$	$\nu\mathbb{C}P^k \ominus \mathbb{C}\nu M$	$2k-2$
$-2\cot(2r)$	$\mathcal{F}$	1

4. $S\phi = \phi S$.
5. The Reeb flow on M is an isometric flow.

Proposition 3.1. (Berndt and Suh, IJM., 2013)

Let M be the tube of radius $0 < r < \pi/2$ around the totally geodesic $\mathbb{C}P^k$ in Q^{2k}. Then the following hold:

1. M is a Hopf hypersurface.
2. The normal bundle of M consists of $\mathfrak{A}$-isotropic singular.
3. M has four distinct constant principal curvatures.

principal curvature	eigenspace	multiplicity
0	$\mathcal{C} \ominus \mathcal{Q}$	2
$\tan(r)$	$T\mathbb{C}P^k \ominus (\mathcal{C} \ominus \mathcal{Q})$	$2k-2$
$-\cot(r)$	$\nu\mathbb{C}P^k \ominus \mathbb{C}\nu M$	$2k-2$
$-2\cot(2r)$	$\mathcal{F}$	1

4. $S\phi = \phi S$.
5. The Reeb flow on M is an isometric flow.

Proposition 3.1. (Berndt and Suh, IJM., 2013)

Let M be the tube of radius $0 < r < \pi/2$ around the totally geodesic $\mathbb{C}P^k$ in Q^{2k}. Then the following hold:

1. M is a Hopf hypersurface.
2. The normal bundle of M consists of $\mathfrak{A}$-isotropic singular.
3. M has four distinct constant principal curvatures.

principal curvature	eigenspace	multiplicity
0	$\mathcal{C} \ominus \mathcal{Q}$	2
$\tan(r)$	$T\mathbb{C}P^k \ominus (\mathcal{C} \ominus \mathcal{Q})$	$2k-2$
$-\cot(r)$	$\nu\mathbb{C}P^k \ominus \mathbb{C}\nu M$	$2k-2$
$-2\cot(2r)$	$\mathcal{F}$	1

4. $S\phi = \phi S$.
5. The Reeb flow on M is an isometric flow.

In this talk we present the classification for the complex quadric $Q^m = SO(m+2)/SO(2)SO(m)$. In view of the previous two results a natural expectation would be that the corresponding classification would lead to the totally geodesic $Q^{m-1} \subset Q^m$. Surprisingly, this is not the case. In fact, we prove

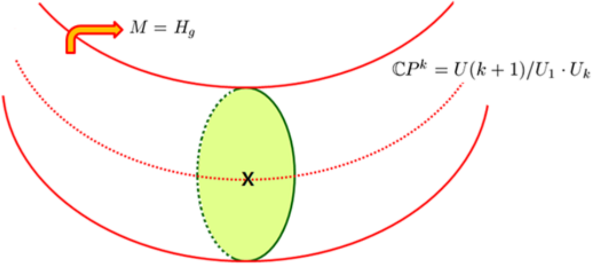
Theorem 3.1. (Berndt and Suh, IJM., 2013)

Let M be a real hypersurface of the complex quadric Q^m, $m \geq 3$. The Reeb flow on M is isometric if and only if m is even, say $m = 2k$, and M is an open part of a tube around a totally geodesic $\mathbb{C}P^k \subset Q^{2k}$.

Complex Quadrics

$\boxed{3}$ In Q^{2k} : Berndt and Suh

M : an open part of a tube over $\mathbb{C}P^k$ in Q^{2k}
H : $H = U(k+1) \hookrightarrow SO(2k+2)$
: Isometry group acting cohomogeneity one



Outline of the Proof of Theorem 3.1

In the following we will investigate real hypersurfaces in Q^m for which the Reeb flow is isometric. From this, we get a complete expression for the covariant derivative as follows:

$$
\begin{aligned}
(\nabla_X S)Y &= \{d\alpha(X)\eta(Y) + g((\alpha S\phi - S^2\phi)X, Y) \\
&+\delta\eta(Y)\rho(X) + \delta g(BX, \phi Y) + \eta(BX)\rho(Y)\}\xi \\
&+\{\eta(Y)\rho(X) + g(BX, \phi Y)\}B\xi + g(BX, Y)\phi B\xi \\
&-\rho(Y)BX - \eta(Y)\phi X - \eta(BY)\phi BX.
\end{aligned}
$$

Lemma 3.1.4

Let M be a real hypersurface in Q^m, $m \geq 3$, with isometric Reeb flow. Then the normal vector field N is $\mathfrak{A}$-isotropic everywhere.

From Proposition and Lemma the principal curvature function α is constant. Then we get

$$(\lambda^2 - \alpha\lambda)Y + (\lambda^2 - \alpha\lambda)Z = (S^2 - \alpha S)X = Y.$$

By virtue of this equation, we can assert the following propositions:

Proposition 3.2

Let M be a real hypersurface in Q^m, $m \geq 3$, with isometric Reeb flow. Then the distributions $\mathcal{Q}$ and $\mathcal{C} \ominus \mathcal{Q} = [B\xi]$ are invariant.

Proposition 3.3

Let M be a real hypersurface in Q^m, $m \geq 3$, with isometric Reeb flow. Then m is even, say $m = 2k$, and the real structure A maps T_λ onto T_μ, and vice versa.

From Proposition and Lemma the principal curvature function α is constant. Then we get

$$(\lambda^2 - \alpha\lambda)Y + (\lambda^2 - \alpha\lambda)Z = (S^2 - \alpha S)X = Y.$$

By virtue of this equation, we can assert the following propositions:

Proposition 3.2

Let M be a real hypersurface in Q^m, $m \geq 3$, with isometric Reeb flow. Then the distributions $\mathcal{Q}$ and $\mathcal{C} \ominus \mathcal{Q} = [B\xi]$ are invariant.

Proposition 3.3

Let M be a real hypersurface in Q^m, $m \geq 3$, with isometric Reeb flow. Then m is even, say $m = 2k$, and the real structure A maps T_λ onto T_μ, and vice versa.

From Proposition and Lemma the principal curvature function α is constant. Then we get

$$(\lambda^2 - \alpha\lambda)Y + (\lambda^2 - \alpha\lambda)Z = (S^2 - \alpha S)X = Y.$$

By virtue of this equation, we can assert the following propositions:

Proposition 3.2

Let M be a real hypersurface in Q^m, $m \geq 3$, with isometric Reeb flow. Then the distributions $\mathcal{Q}$ and $\mathcal{C} \ominus \mathcal{Q} = [B\xi]$ are invariant.

Proposition 3.3

Let M be a real hypersurface in Q^m, $m \geq 3$, with isometric Reeb flow. Then m is even, say $m = 2k$, and the real structure A maps T_λ onto T_μ, and vice versa.

Key Points of the Proof

For each point $[z] \in M$ we denote by $\gamma_{[z]}$ the geodesic in Q^{2k} with $\gamma_{[z]}(0) = [z]$ and $\dot{\gamma}_{[z]}(0) = N_{[z]}$ and by F the smooth map

$$F : M \longrightarrow Q^m, [z] \longrightarrow \gamma_{[z]}(r).$$

F is the displacement of M at distance r in the direction of N. Thee differential $d_{[z]}F$ of F at $[z]$ can be computed by

$$d_{[z]}F(X) = Z_X(r),$$

where Z_X is the Jacobi vector field along $\gamma_{[z]}$ with $Z_X(0) = X$ and $Z_X'(0) = -SX$. The $\mathfrak{A}$-isotropic N gives that $R_N = R(Z, N)N$ has the three constant eigenvalues $0, 1, 4$ with corresponding eigenbundles

$$\nu M \oplus (\mathcal{C} \ominus \mathcal{Q}) = \nu M \oplus T_\nu,$$

$$\mathcal{Q} = T_\lambda \oplus T_\mu \quad \text{and} \quad \mathcal{F} = \mathrm{T}_\alpha.$$

Rigidity of totally geodesic submanifolds now implies that the entire submanifold M is an open part of a tube of radius r around a k-dimensional connected, complete, totally geodesic complex submanifold P of Q^{2k}.

According to Klein's classification the submanifold P is either $Q^k \subset Q^{2k}$ ($\mathfrak{A}$-invariant) or $\mathbb{C}P^k \subset Q^{2k}$ ($\mathfrak{A}$-isotropic). But we have proved that the normal vector N is $\mathfrak{A}$-isotropic. Then it follows that M is congruent to an open part of a tube around $\mathbb{C}P^k$.

This concludes the proof of our Theorem 3.1 for real hypersurfaces with isometric Reeb flow in complex quadric.

Some Key Propositions

A contact hypersurface in a Kaehler manifold is a real hypersurface satisfying the condition:

$$S\phi + \phi S = k\phi, \quad k = 2\rho \neq 0 : \quad \text{constant}$$

Then we can apply its result to give a classification in Q^m as follows:

Proposition 3.2.1. (Berndt and Suh, Proc. AMS., 2015)

The following statements are equivalent:

- (i) The function α is constant,
- (ii) M has constant mean curvature,
- (iii) JN is an eigenvector of the normal Jacobi $\bar{R}_N$.

Some Key Propositions

A contact hypersurface in a Kaehler manifold is a real hypersurface satisfying the condition:

$$S\phi + \phi S = k\phi, \quad k = 2\rho \neq 0 : \quad \text{constant}$$

Then we can apply its result to give a classification in Q^m as follows:

Proposition 3.2.1. (Berndt and Suh, Proc. AMS., 2015)

The following statements are equivalent:

- (i) The function α is constant,
- (ii) M has constant mean curvature,
- (iii) JN is an eigenvector of the normal Jacobi $\bar{R}_N$.

Some Key Propositions

A contact hypersurface in a Kaehler manifold is a real hypersurface satisfying the condition:

$$S\phi + \phi S = k\phi, \quad k = 2\rho \neq 0 : \quad \text{constant}$$

Then we can apply its result to give a classification in Q^m as follows:

Proposition 3.2.1. (Berndt and Suh, Proc. AMS., 2015)

The following statements are equivalent:

- (i) The function α is constant,
- (ii) M has constant mean curvature,
- (iii) JN is an eigenvector of the normal Jacobi $\bar{R}_N$.

Some Key Propositions

A contact hypersurface in a Kaehler manifold is a real hypersurface satisfying the condition:

$$S\phi + \phi S = k\phi, \quad k = 2\rho \neq 0 : \quad \text{constant}$$

Then we can apply its result to give a classification in Q^m as follows:

Proposition 3.2.1. (Berndt and Suh, Proc. AMS., 2015)

The following statements are equivalent:

- (i) The function α is constant,
- (ii) M has constant mean curvature,
- (iii) JN is an eigenvector of the normal Jacobi $\bar{R}_N$.

Some Key Propositions

Proposition 3.2.2

Let M be a contact hypersurfaces of the complex quadric Q^m (resp. Q^{m*}), $m \geq 3$. Then the following statements are equivalent:

- (i) JN is an eigenvector of the normal Jacobi operator $\bar{R}_N = \bar{R}(\cdot, N)N$ everywhere ,
- (ii) N is $\mathfrak{A}$-principal or $\mathfrak{A}$-isotropic everywhere,
- (iii) The normal vector N is singular in Q^m (resp. in Q^{m*}).

Proposition 3.2.3

Let M be a contact hypersurface in Q^m (resp. in Q^{m*}). Then the normal vector N can not be $\mathfrak{A}$-isotropic.

Some Key Propositions

Proposition 3.2.2

Let M be a contact hypersurfaces of the complex quadric Q^m (resp. Q^{m*}), $m \geq 3$. Then the following statements are equivalent:

- (i) JN is an eigenvector of the normal Jacobi operator $\bar{R}_N = \bar{R}(\cdot, N)N$ everywhere ,
- (ii) N is $\mathfrak{A}$-principal or $\mathfrak{A}$-isotropic everywhere,
- (iii) The normal vector N is singular in Q^m (resp. in Q^{m*}).

Proposition 3.2.3

Let M be a contact hypersurface in Q^m (resp. in Q^{m*}). Then the normal vector N can not be $\mathfrak{A}$-isotropic.

Some Key Propositions

Proposition 3.2.2

Let M be a contact hypersurfaces of the complex quadric Q^m (resp. Q^{m*}), $m \geq 3$. Then the following statements are equivalent:

- (i) JN is an eigenvector of the normal Jacobi operator $\bar{R}_N = \bar{R}(\cdot, N)N$ everywhere ,
- (ii)N is $\mathfrak{A}$-principal or $\mathfrak{A}$-isotropic everywhere,
- (iii)The normal vector N is singular in Q^m (resp. in Q^{m*}).

Proposition 3.2.3

Let M be a contact hypersurface in Q^m (resp. in Q^{m*}). Then the normal vector N can not be $\mathfrak{A}$-isotropic.

Some Key Propositions

Proposition 3.2.2

Let M be a contact hypersurfaces of the complex quadric Q^m (resp. Q^{m*}), $m \geq 3$. Then the following statements are equivalent:

- (i) JN is an eigenvector of the normal Jacobi operator $\bar{R}_N = \bar{R}(\cdot, N)N$ everywhere ,
- (ii) N is $\mathfrak{A}$-principal or $\mathfrak{A}$-isotropic everywhere,
- (iii) The normal vector N is singular in Q^m (resp. in Q^{m*}).

Proposition 3.2.3

Let M be a contact hypersurface in Q^m (resp. in Q^{m*}). Then the normal vector N can not be $\mathfrak{A}$-isotropic.

Some Key Propositions

Proposition 3.2.2

Let M be a contact hypersurfaces of the complex quadric Q^m (resp. Q^{m*}), $m \geq 3$. Then the following statements are equivalent:

- (i) JN is an eigenvector of the normal Jacobi operator $\bar{R}_N = \bar{R}(\cdot, N)N$ everywhere ,
- (ii)N is $\mathfrak{A}$-principal or $\mathfrak{A}$-isotropic everywhere,
- (iii)The normal vector N is singular in Q^m (resp. in Q^{m*}).

Proposition 3.2.3

Let M be a contact hypersurface in Q^m (resp. in Q^{m*}). Then the normal vector N can not be $\mathfrak{A}$-isotropic.

Contact hypersurfaces in complex quadrics

By viture of key Propositions and some remarks mentioned above, we give a classification in Q^m as follows:

Theorem 3.2. (Berndt and Suh, Proc. AMS., 2015)

Let M be a connected real hypersurface with constant mean curvature in complex quadric Q^m, $m \geq 3$. Then M is contact if and only if M is an open part of a tube of radius $0 < r < \frac{\pi}{2\sqrt{2}}$ around the sphere S^m embedded in Q^m.

Contact hypersurfaces in noncompact quadrics

Moreover, we give a classification of contact hypersurfaces in $Q^{m*} = SO^0_{m,2}/SO_mSO_2$ for $m \geq 3$ as follows:

Theorem 3.3. (Berndt and Suh, Proc. AMS., 2015)

Let M be a connected real hypersurface with cmc in Q^{m*}, $m \geq 3$. Then M is contact if and only if M is an open part of one of the following

- (i)the tube of radius $r \in \mathbf{R}_+$ around a totally geodesic $Q^{(m-1)*}$ which is embedded in Q^{m*},
- (ii)the tube of radius $r \in \mathbf{R}_+$ around a totally real totally geodesic $\mathbf{R}H^m$ embedded in Q^{m*} as a real form of Q^{m*}.
- (iii) a horosphere in Q^{m*} whose center at infinity is the equivalence class of an $\mathfrak{A}$-principal geodesic in Q^{m*}.

Contact hypersurfaces in noncompact quadrics

Moreover, we give a classification of contact hypersurfaces in $Q^{m*} = SO^0_{m,2}/SO_mSO_2$ for $m \geq 3$ as follows:

Theorem 3.3. (Berndt and Suh, Proc. AMS., 2015)

Let M be a connected real hypersurface with cmc in Q^{m*}, $m \geq 3$. Then M is contact if and only if M is an open part of one of the following

- (i)the tube of radius $r \in \mathbf{R}_+$ around a totally geodesic $Q^{(m-1)*}$ which is embedded in Q^{m*},
- (ii)the tube of radius $r \in \mathbf{R}_+$ around a totally real totally geodesic $\mathbf{R}H^m$ embedded in Q^{m*} as a real form of Q^{m*}.
- (iii) a horosphere in Q^{m*} whose center at infinity is the equivalence class of an $\mathfrak{A}$-principal geodesic in Q^{m*}.

Contact hypersurfaces in noncompact quadrics

Moreover, we give a classification of contact hypersurfaces in $Q^{m*} = SO^0_{m,2}/SO_mSO_2$ for $m \geq 3$ as follows:

Theorem 3.3. (Berndt and Suh, Proc. AMS., 2015)

Let M be a connected real hypersurface with cmc in Q^{m*}, $m \geq 3$. Then M is contact if and only if M is an open part of one of the following

- (i)the tube of radius $r \in \mathbf{R}_+$ around a totally geodesic $Q^{(m-1)*}$ which is embedded in Q^{m*},
- (ii)the tube of radius $r \in \mathbf{R}_+$ around a totally real totally geodesic $\mathbf{R}H^m$ embedded in Q^{m*} as a real form of Q^{m*}.
- (iii) a horosphere in Q^{m*} whose center at infinity is the equivalence class of an $\mathfrak{A}$-principal geodesic in Q^{m*}.

Contact hypersurfaces in noncompact quadrics

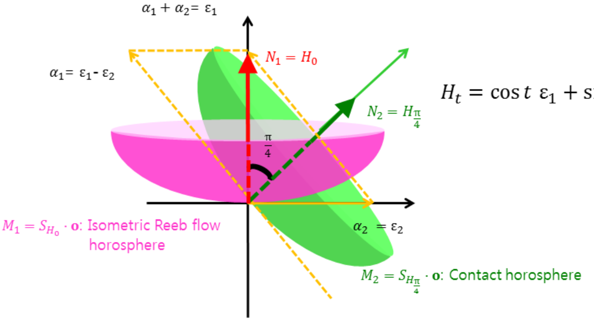
Moreover, we give a classification of contact hypersurfaces in $Q^{m*} = SO^0_{m,2}/SO_m SO_2$ for $m \geq 3$ as follows:

Theorem 3.3. (Berndt and Suh, Proc. AMS., 2015)

Let M be a connected real hypersurface with cmc in Q^{m*}, $m \geq 3$. Then M is contact if and only if M is an open part of one of the following

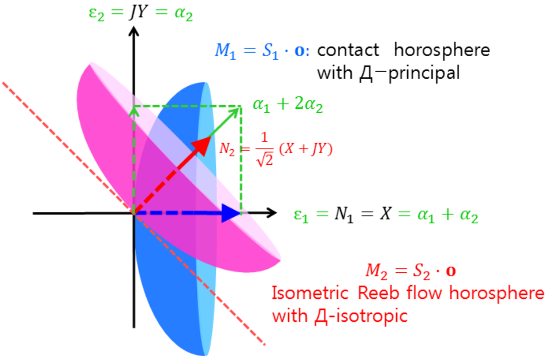
- (i)the tube of radius $r \in \mathbf{R}_+$ around a totally geodesic $Q^{(m-1)*}$ which is embedded in Q^{m*},
- (ii)the tube of radius $r \in \mathbf{R}_+$ around a totally real totally geodesic $\mathbf{R}H^m$ embedded in Q^{m*} as a real form of Q^{m*}.
- (iii) a horosphere in Q^{m*} whose center at infinity is the equivalence class of an $\mathfrak{A}$-principal geodesic in Q^{m*}.

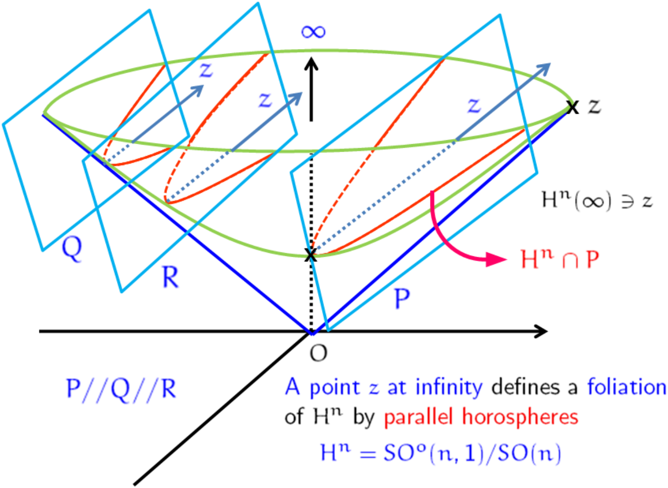
Horosphers in Complex Hyperbolic Grassmannians



$$H_t = \cos t\ \varepsilon_1 + \sin t\ \varepsilon_2 \in Д$$

Horosphers in Complex Hyperbolic Quadrics





A point z at infinity defines a foliation of H^n by parallel horospheres

$$H^n = SO^o(n,1)/SO(n)$$

Codazzi type hypersurfaces

When the shape operator S of M in Q^m satisfying $(\nabla_X S)Y = (\nabla_Y S)X$ for any X, Y on M in Q^m, we say that the shape operator is of *Codazzi type*.

Theorem 3.4.1. (Suh, IJM., 2014)

There do not exist any real hypersurfaces in complex quadric Q^m, $m \geq 3$, with shape operator of Codazzi type.

Theorem 3.4.2. (Suh, IJM., 2014)

There do not exist any real hypersurfaces in complex quadric Q^m, $m \geq 3$, with parallel shape operator.

Codazzi type hypersurfaces

When the shape operator S of M in Q^m satisfying $(\nabla_X S)Y = (\nabla_Y S)X$ for any X, Y on M in Q^m, we say that the shape operator is of *Codazzi type*.

Theorem 3.4.1. (Suh, IJM., 2014)

There do not exist any real hypersurfaces in complex quadric Q^m, $m \geq 3$, with shape operator of Codazzi type.

Theorem 3.4.2. (Suh, IJM., 2014)

There do not exist any real hypersurfaces in complex quadric Q^m, $m \geq 3$, with parallel shape operator.

Ricci Parallel Hypersurfaces

Now we consider the notion of *Ricci parallelism* for hypersurfaces in Q^m , that is, $\nabla Ric = 0$.

In this section we consider only an $\mathfrak{A}$-principal normal vector field N, that is, $AN = N$. Then

$$Ric(Y) = (2m-1)Y - 2\eta(Y)\xi - AY + hSY - S^2Y,$$

where $h = trS$ denotes the mean curvature and is defined by the trace of the shape operator S of M in Q^m.
Then from this, by the parallel Ricci tensor, we have

$$\begin{aligned} 0 &= -2g(\phi SX, Y)\xi - 2\eta(Y)\phi SX - (\nabla_X A)Y + (Xh)SY \\ &\quad + h(\nabla_X S)Y - (\nabla_X S^2)Y, \end{aligned}$$

where $(\nabla_X A)Y = \nabla_X(AY) - A\nabla_X Y$. Here, AY belongs to T_zM, $z \in M$.

Theorem 3.5. (Suh, Adv. in Math., 2015)

There does not exist any Hopf hypersurfaces in the complex quadric Q^m with parallel Ricci tensor and $\mathfrak{A}$-principal normal vector field.

We consider a maximal $\mathfrak{A}$-invariant subspace $\mathcal{Q}_z$ of T_zM, $z \in M$, defined by

$$\mathcal{Q}_z = \{X \in T_zM \mid AX \in T_zM \text{ for all } A \in \mathfrak{A}_z\}$$

Then the orthogonal complement of $\mathcal{Q}$ in $\mathcal{C}$, becomes $\mathcal{Q}_z^{\perp} = \mathrm{Span}\{\mathrm{A}\xi, \mathrm{AN}\}$.

Ricci Parallel Hypersurfaces II

For real hypersurfaces with parallel Ricci tensor and $\mathfrak{A}$-isotropic unit normal we have the following

$$\begin{aligned}(\nabla_Y Ric)X &= \nabla_Y(Ric(X)) - Ric(\nabla_Y X)\\ &= -3(\nabla_Y\eta)(X)\xi - 3\eta(X)\nabla_Y\xi\\ &\quad +g(X,\nabla_Y(AN))AN - g(AX,N)\nabla_Y(AN)\\ &\quad +g((\nabla_Y(A\xi),X)A\xi + \eta(AX)\nabla_Y(A\xi) + (Yh)SX\\ &\quad +h(\nabla_Y S)X - (\nabla_Y S^2)X,\end{aligned}$$

where we have used the following for $\mathfrak{A}$-isotropic unit normal

$$g(\xi, A\xi) = 0, g(\xi, AN) = 0, \quad \text{and} \quad \mathrm{g(AN,N)} = 0.$$

Then motivated by the above result, we give another theorem in the complex quadric Q^m with parallel Ricci tensor and $\mathfrak{A}$-isotropic unit normal as follows:

Theorem 3.6. (Suh, Adv. in Math., 2015)

Let M be a Hopf real hypersurface in the complex quadric Q^m, $m \geq 4$, with parallel Ricci tensor and $\mathfrak{A}$-isotropic unit normal N. If the shape operator commutes with the structure tensor on the distribution $\mathcal{Q}^{\perp}$, then M has 3 distinct constant principal curvatures which are given by

$$\alpha = \sqrt{\frac{2m-1}{2}}, \gamma(=\alpha) = \sqrt{\frac{2m-1}{2}}, \lambda = 0, \mu = -\frac{2\sqrt{2}}{\sqrt{2m-1}},$$

with corresponding principal curvature spaces

Harmonic Curvature I

$T_\alpha = [\xi], T_\gamma = [A\xi, AN], \phi(T_\lambda) = T_\mu, \mathrm{dim}\ \mathrm{T}_\lambda = \mathrm{dim}\ \mathrm{T}_\mu = \mathrm{m} - 2,$ *respectively*.

We consider the notion of *harmonic curvature* for hypersurfaces M in Q^m , that is, $(\nabla_X Ric)Y = (\nabla_Y Ric)X$ for any vector fields X and Y on M in Q^m.
It is equivalent to $\delta R = 0$ and $dR = 0$(*the second Bianchi identity*) for the curvature tensor $R(X, Y)Z$ of M in Q^m.

Then for hypersurfaces in Q^m with $\mathfrak{A}$-principal normal we assert the following

Theorem 3.7. (Suh, JMPA 2016)

Let M be a Hopf real hypersurface in the complex quadric Q^m, $m \geq 4$, with harmonic curvature. If the unit norml N is $\mathfrak{A}$-principal, then M has at most 5 distinct constant principal curvatures, five of which are given by

$$\alpha, \quad \lambda_1, \quad \mu_1, \quad \lambda_2, \quad \text{and} \quad \mu_2$$

with corresponding principal curvature spaces:

$T_\alpha = [\xi], \phi T_{\lambda_1} = T_{\mu_1}, \phi T_{\lambda_2} = T_{\mu_2}, \dim \mathrm{T}_{\lambda_1} + \dim \mathrm{T}_{\lambda_2} = \mathrm{m} - 1, \dim \mathrm{T}_{\mu_1} + \dim \mathrm{T}_{\mu_2} = \mathrm{m} - 1.$

Here four roots λ_i and μ_i, $i = 1, 2$ satisfy the two kinds of quadratic equation that

$$2x^2 - 2\beta x + 2 + \alpha\beta = 0,$$

where the function β is denoted by $\beta = \frac{\alpha^2+1\pm\sqrt{(\alpha^2+1)^2+4\alpha h}}{\alpha}$.

Harmonic Curvature II

Theorem 3.8. (Suh, JMPA 2016)

Let M be a Hopf real hypersurface in the complex quadric Q^m, $m \geq 4$, with harmonic curvature and $\mathfrak{A}$-isotropic unit normal N. If the shape operator commutes with the structure tensor on the distribution $\mathcal{Q}^{\perp}$, then $M \approx$ a tube around $\mathbf{C}P^k \subset Q^m$, $m = 2k$, or M has at most 6 distinct constant principal curvatures given by

$$\alpha, \quad \gamma = 0(\alpha), \quad \lambda_1, \quad \mu_1, \quad \lambda_2 \quad \text{and} \quad \mu_2$$

with corresponding principal curvature spaces

$$T_\alpha = [\xi], T_\gamma = [A\xi, AN], \phi(T_{\lambda_1}) = T_{\mu_1}, \ \phi T_{\lambda_2} = T_{\mu_2}.$$

$$\dim \mathrm{T}_{\lambda_1} + \dim \mathrm{T}_{\lambda_2} = \mathrm{m} - 2, \dim \mathrm{T}_{\mu_1} + \dim \mathrm{T}_{\mu_2} = \mathrm{m} - 2.$$

Here four roots λ_i and μ_i, $i = 1, 2$ satisfy the equation that

$$2x^2 - 2\beta x + 2 + \alpha\beta = 0,$$

where the function β denotes $\beta = \frac{\alpha^2+2\pm\sqrt{(\alpha^2+2)^2+4\alpha h}}{\alpha}$. In particular, $\alpha = \sqrt{\frac{2m-1}{2}}$, $\gamma(= \alpha) = \sqrt{\frac{2m-1}{2}}$, $\lambda = 0$, $\mu = -\frac{2\sqrt{2}}{\sqrt{2m-1}}$, with multiplicities 1,2,$m-2$ and $m-2$ respectively.

Finally, we want to mention the following problems:

Problem 4

How can we derive the fact that the unit normal N of M in Q^m (or Q^{m*}) is $\mathfrak{A}$-principal or $\mathfrak{A}$-isotropic, if M is assumed with parallel Ricci tensor or harmonic curvature ?

Problem 5

If M is a real hypersurface in the complex dual quadric Q^{m*} with parallel Ricci tensor, what can we say about them ?

Problem 6

If M is a real hypersurface in the complex dual quadric Q^{m*} with harmonic curvature, what can we say about them ?

Finally, we want to mention the following problems:

Problem 4

How can we derive the fact that the unit normal N of M in Q^m (or Q^{m*}) is $\mathfrak{A}$-principal or $\mathfrak{A}$-isotropic, if M is assumed with parallel Ricci tensor or harmonic curvature ?

Problem 5

If M is a real hypersurface in the complex dual quadric Q^{m*} with parallel Ricci tensor, what can we say about them ?

Problem 6

If M is a real hypersurface in the complex dual quadric Q^{m*} with harmonic curvature, what can we say about them ?

Finally, we want to mention the following problems:

Problem 4

How can we derive the fact that the unit normal N of M in Q^m (or Q^{m*}) is $\mathfrak{A}$-principal or $\mathfrak{A}$-isotropic, if M is assumed with parallel Ricci tensor or harmonic curvature ?

Problem 5

If M is a real hypersurface in the complex dual quadric Q^{m*} with parallel Ricci tensor, what can we say about them ?

Problem 6

If M is a real hypersurface in the complex dual quadric Q^{m*} with harmonic curvature, what can we say about them ?

Pseudo Einstein in $G_2(\mathbb{C}^{m+2})$

A real hypersurface M in $G_2(\mathbb{C}^{m+2})$ is said to be *pseudo-Einstein* if the Ricci tensor Ric of M satisfies

$$Ric(X) = aX + b\eta(X)\xi + c\sum_{i=1}^{3}\eta_i(X)\xi_i$$

for any constants a, b and c on M.

Theorem 4.1. (PSW, 2010 JGP)

Let M be a pseudo-Einstein Hopf in $G_2(\mathbf{C}^{m+2})$. Then $M \approx$
(a) a tube of r, $\cot^2\sqrt{2}r = \frac{m-1}{2}$, over $G_2(\mathbf{C}^{m+1})$, where $a = 4m+8$, $b+c = -2(m+1)$, provided that $c \neq -4$.
(b) a tube of r, $\cot r = \frac{1+\sqrt{4m-3}}{2(m-1)}$, over $\mathbf{H}P^m$, $m = 2n$, where $a = 8n+6$, $b = -16n+10$, $c = -2$.

Pseudo Einstein in $SU_{2,m}/S(U_2U_m)$

A real hypersurface M in $SU_{2,m}/S(U_2U_m)$ is said to be *pseudo-Einstein* if the Ricci tensor Ric of M satisfies

$$Ric(X) = aX + b\eta(X)\xi + c\sum_{i=1}^{3}\eta_i(X)\xi_i$$

for any constants a, b and c on M.

Theorem 4.2. (Suh, 2016 AAM)

Let M be a pseudo-Einstein Hopf in $SU_{2,m}/S(U_2U_m)$, $m \geq 2$. Then $M \approx$ a hypersurface with four curvatures $\sqrt{2}$, 0, $\lambda = \frac{1}{\sqrt{2}}$ and $\mu = \frac{q-4m+3}{q\sqrt{2}}$ such that $p + q = 4(m-2)$, where p and q denote the multiplicities of λ and μ. In this case $M \approx$ a proper pseudo-Einstein with $a = -\frac{1}{2}(4m+5)$, $b = c = \frac{3}{2}$.

Pseudo Einstein in Q^m

A real hypersurface M in Q^m is said to be *pseudo-Einstein* if the Ricci tensor Ric of M satisfies

$$Ric(X) = aX + b\eta(X)\xi$$

for any constants a and b on M.

Theorem 4.3. (Suh, Submitted)

Let M be a pseudo-Einstein Hopf in Q^m, $m \geq 3$. Then
(i) $M \approx$ a tube of r over a tot. real and tot. geodesic m-dim. S^m in Q^m, with $a = 2m$, and $b = -2m$.
(ii) $m = 2k$, $M \approx$ a tube of r, $r = \cot^{-1}\sqrt{\frac{k}{k-1}}$ over a tot. geodesic k-dim. $\mathbb{C}P^k$ in Q^{2k} with $a = 4k$ and $b = -4 + \frac{2}{k}$.

Now we consider an Einstein hypersurface in Q^m. Then the Ricci tensor of M becomes $Ric = \lambda g$. In case (i) in above Theorem 4.3, there do not exist any Einstein hypersurfaces in Q^m, because $b = -2m$ is non-vanishing. In this case, the unit normal N is $\mathfrak{A}$-principal.

Moreover, in (ii), if M is assumed to be Einstein, then the constant should be $b = 0$. This gives $4 = \frac{2}{k}$, which implies a contradiction. In this case M has an $\mathfrak{A}$-isotropic.

Corollary 4.4.

There do not exist any Einstein Hopf real hypersurfaces in the complex quadric Q^m, $m \geq 3$.

Pseudo-anti commuting and Ricci soliton

We consider a new notion of *pseudo-anti commuting* Ricci tensor which is defined by

$$Ric \cdot \phi + \phi \cdot Ric = \kappa \phi, \quad \kappa \neq 0 : \text{constant},$$

where the structure tensor ϕ is induced from the Kähler structure J of Hermitian symmetric space.

Theorem 4.5. (Suh, Submitted)

Let M be a pseudo-anti commuting Hopf in Q^m, $m \geq 3$. Then
(i) $M \approx$ a tube of r, $0 < r < \frac{\pi}{2\sqrt{2}}$, around a tot. real and tot. geodesic m-dim. S^m in Q^m, with $\mathfrak{A}$-principal unit normal.
(ii) $M \approx$ a tube of r, $0 < r < \frac{\pi}{2}$, $r \neq \frac{\pi}{4}$, around a tot. geodesic k-dim. $\mathbb{C}P^k$ in Q^{2k}, $m = 2k$, with $\mathfrak{A}$-isotropic.

A solution of the Ricci flow equation $\frac{\partial}{\partial t} g(t) = -2Ric(g(t))$ is given by

$$\frac{1}{2}(\mathfrak{L}_V g)(X, Y) + \mathrm{Ric}(X, Y) = \rho g(X, Y),$$

where ρ is a constant and $\mathfrak{L}_V$ denotes the Lie derivative along the direction of the vector field V (see Hamilton, Morgan and Tian, Perelmann). Then the solution is said to be a *Ricci soliton* with potential vector field V and Ricci soliton constant ρ, and surprisingly, it satisfies the pseudo-anti commuting condition $S\phi + \phi S = \kappa\phi$, where $\kappa = 2\rho$ is non-zero constant.

Theorem 4.6. (Suh, Submitted)

Let (M, g, ξ, ρ) be a Ricci soliton on Hopf real hypersurfaces in Q^m, $m \geq 3$. Then

(i) M is an open part of a tube of radius r around a tot. real and tot. geodesic m-dim. unit sphere S^m in Q^m, with radii $r = \frac{1}{\sqrt{2}} \cot^{-1}\left(\frac{1}{2\sqrt{2(m-1)}}\right)$ and $r = \frac{1}{\sqrt{2}} \cot^{-1}\left(\frac{1}{2\sqrt{2m}}\right)$. Here the unit normal N is $\mathfrak{A}$-principal.

(ii) M is an open part of a tube of radius $r = \tan^{-1}\sqrt{\frac{k}{k-1}}$ around a tot. geodesic k-dim. complex projective space $\mathbb{C}P^k$ in Q^{2k}, $m = 2k$. Here the unit normal N is $\mathfrak{A}$-isotropic.

References

- J. Berndt and Y.J. Suh, *Real hypersurfaces in complex two-plane Grassmannians*, Monatshefte für Math. **127**(1999), 1-14.
- J. Berndt and Y.J. Suh, *Isometric flows on real hypersurfaces in complex two-plane Grassmannians*, Monatshefte für Math. **137**(2002), 87-98.
- S. Montiel and A. Romero, *On some real hypersurfaces of a complex hyperbolic space*, Geom. Dedicata **20**(1986), 245-261.
- M. Okumura, *On some real hypersurfaces of a complex projective space*, Trans. Amer. Math. Soc. **212**(2006), 355-364.

References

- J. Berndt and Y.J. Suh, *Real hypersurfaces in complex two-plane Grassmannians*, Monatshefte für Math. **127**(1999), 1-14.
- J. Berndt and Y.J. Suh, *Isometric flows on real hypersurfaces in complex two-plane Grassmannians*, Monatshefte für Math. **137**(2002), 87-98.
- S. Montiel and A. Romero, *On some real hypersurfaces of a complex hyperbolic space*, Geom. Dedicata **20**(1986), 245-261.
- M. Okumura, *On some real hypersurfaces of a complex projective space*, Trans. Amer. Math. Soc. **212**(2006), 355-364.

References

- J. Berndt and Y.J. Suh, *Real hypersurfaces in complex two-plane Grassmannians*, Monatshefte für Math. **127**(1999), 1-14.
- J. Berndt and Y.J. Suh, *Isometric flows on real hypersurfaces in complex two-plane Grassmannians*, Monatshefte für Math. **137**(2002), 87-98.
- S. Montiel and A. Romero, *On some real hypersurfaces of a complex hyperbolic space*, Geom. Dedicata **20**(1986), 245-261.
- M. Okumura, *On some real hypersurfaces of a complex projective space*, Trans. Amer. Math. Soc. **212**(2006), 355-364.

References

- J. Berndt and Y.J. Suh, *Real hypersurfaces in complex two-plane Grassmannians*, Monatshefte für Math. **127**(1999), 1-14.
- J. Berndt and Y.J. Suh, *Isometric flows on real hypersurfaces in complex two-plane Grassmannians*, Monatshefte für Math. **137**(2002), 87-98.
- S. Montiel and A. Romero, *On some real hypersurfaces of a complex hyperbolic space*, Geom. Dedicata **20**(1986), 245-261.
- M. Okumura, *On some real hypersurfaces of a complex projective space*, Trans. Amer. Math. Soc. **212**(2006), 355-364.

References II

- J.D. Perez and Y.J. Suh, *Real hypersurfaces of quaternionic projective space satisfying* $\nabla_{U_i} R = 0$, Diff. Geom. Appl. **7**(1997), 211-217.
- Y.J. Suh, *Real hypersurfaces in complex two-plane Grassmannians with commuting Ricci tensor*, J. of Geom. and Physics, **60**(2010), 1792-1805.
- Y.J. Suh, *Real hypersurfaces in complex two-plane Grassmannians with parallel Ricci tensor*, Proc. Royal Soc. Edinburgh **142(A)**(2012), 1309-1324.

References II

- J.D. Perez and Y.J. Suh, *Real hypersurfaces of quaternionic projective space satisfying* $\nabla_{U_i} R = 0$, Diff. Geom. Appl. **7**(1997), 211-217.
- Y.J. Suh, *Real hypersurfaces in complex two-plane Grassmannians with commuting Ricci tensor*, J. of Geom. and Physics, **60**(2010), 1792-1805.
- Y.J. Suh, *Real hypersurfaces in complex two-plane Grassmannians with parallel Ricci tensor*, Proc. Royal Soc. Edinburgh **142(A)**(2012), 1309-1324.

References II

- J.D. Perez and Y.J. Suh, *Real hypersurfaces of quaternionic projective space satisfying* $\nabla_{U_i} R = 0$, Diff. Geom. Appl. **7**(1997), 211-217.
- Y.J. Suh, *Real hypersurfaces in complex two-plane Grassmannians with commuting Ricci tensor*, J. of Geom. and Physics, **60**(2010), 1792-1805.
- Y.J. Suh, *Real hypersurfaces in complex two-plane Grassmannians with parallel Ricci tensor*, Proc. Royal Soc. Edinburgh **142(A)**(2012), 1309-1324.

References III

- Y.J. Suh, *Real hypersurfaces in complex two-plane Grassmannians with Reeb parallel Ricci tensor*, J. of Geom. and Physics, **64**(2013), 1-11.
- Y.J. Suh, *Real hypersurfaces in complex two-plane Grassmannians with harmonic curvature*, Journal de Math. Pures Appl., **100**(2013), 16-33.
- Y.J. Suh, *Hypersurfaces with isometric Reeb flow in complex hyperbolic two-plane Grassmannians*, Advances in Applied Mathematics, **50**(2013), 645-659.
- Y.J. Suh and C. Woo, *Real hypersurfaces in complex hyperbolic two-plane Grassmannians with parallel Ricci tensor*, Math. Nachr., **287**(2014), 1524-1529.

References III

- Y.J. Suh, *Real hypersurfaces in complex two-plane Grassmannians with Reeb parallel Ricci tensor*, J. of Geom. and Physics, **64**(2013), 1-11.
- Y.J. Suh, *Real hypersurfaces in complex two-plane Grassmannians with harmonic curvature*, Journal de Math. Pures Appl., **100**(2013), 16-33.
- Y.J. Suh, *Hypersurfaces with isometric Reeb flow in complex hyperbolic two-plane Grassmannians*, Advances in Applied Mathematics, **50**(2013), 645-659.
- Y.J. Suh and C. Woo, *Real hypersurfaces in complex hyperbolic two-plane Grassmannians with parallel Ricci tensor*, Math. Nachr., **287**(2014), 1524-1529.

References III

- Y.J. Suh, *Real hypersurfaces in complex two-plane Grassmannians with Reeb parallel Ricci tensor*, J. of Geom. and Physics, **64**(2013), 1-11.
- Y.J. Suh, *Real hypersurfaces in complex two-plane Grassmannians with harmonic curvature*, Journal de Math. Pures Appl., **100**(2013), 16-33.
- Y.J. Suh, *Hypersurfaces with isometric Reeb flow in complex hyperbolic two-plane Grassmannians*, Advances in Applied Mathematics, **50**(2013), 645-659.
- Y.J. Suh and C. Woo, *Real hypersurfaces in complex hyperbolic two-plane Grassmannians with parallel Ricci tensor*, Math. Nachr., **287**(2014), 1524-1529.

References III

- Y.J. Suh, *Real hypersurfaces in complex two-plane Grassmannians with Reeb parallel Ricci tensor*, J. of Geom. and Physics, **64**(2013), 1-11.
- Y.J. Suh, *Real hypersurfaces in complex two-plane Grassmannians with harmonic curvature*, Journal de Math. Pures Appl., **100**(2013), 16-33.
- Y.J. Suh, *Hypersurfaces with isometric Reeb flow in complex hyperbolic two-plane Grassmannians*, Advances in Applied Mathematics, **50**(2013), 645-659.
- Y.J. Suh and C. Woo, *Real hypersurfaces in complex hyperbolic two-plane Grassmannians with parallel Ricci tensor*, Math. Nachr., **287**(2014), 1524-1529.

References IV

- Y.J. Suh, *Real hypersurfaces in the complex quadric with Reeb parallel shape operator*, International J. Math., **25**(2014), 1450059(17 pp).
- Y.J. Suh, *Real hypersurfaces in complex hyperbolic two-plane Grassmannians with commuting Ricci tensor*, Int. J. Math., **26**(2015), 1550008 (26 pages).
- Y.J. Suh, *Real hypersurfaces in the complex quadric with parallel Ricci tensor*, Advances in Mathematics, **281**(2015), 886-905.

References IV

- Y.J. Suh, *Real hypersurfaces in the complex quadric with Reeb parallel shape operator*, International J. Math., **25**(2014), 1450059(17 pp).
- Y.J. Suh, *Real hypersurfaces in complex hyperbolic two-plane Grassmannians with commuting Ricci tensor*, Int. J. Math., **26**(2015), 1550008 (26 pages).
- Y.J. Suh, *Real hypersurfaces in the complex quadric with parallel Ricci tensor*, Advances in Mathematics, **281**(2015), 886-905.

References IV

- Y.J. Suh, *Real hypersurfaces in the complex quadric with Reeb parallel shape operator*, International J. Math., **25**(2014), 1450059(17 pp).
- Y.J. Suh, *Real hypersurfaces in complex hyperbolic two-plane Grassmannians with commuting Ricci tensor*, Int. J. Math., **26**(2015), 1550008 (26 pages).
- Y.J. Suh, *Real hypersurfaces in the complex quadric with parallel Ricci tensor*, Advances in Mathematics, **281**(2015), 886-905.

References V

- Y.J. Suh, *Real hypersurfaces in the complex quadric with harmonic curvature*, Journal de Math. Pures Appl., **103**(2016), in press.
- Y.J. Suh, *Pseudo-Einstein real hypersurfaces in complex hyperbolic two-plane Grassmannians*, Advances in Appl. Math., **57**(2016), in press.
- Y.J. Suh, *Pseudo-Einstein real hypersurfaces in the complex quadric*, submitted.
- Y.J. Suh, *Pseudo-anti commuting and Ricci soliton real hypersurfaces in the complex quadric*, submitted.

References V

- Y.J. Suh, *Real hypersurfaces in the complex quadric with harmonic curvature*, Journal de Math. Pures Appl., **103**(2016), in press.
- Y.J. Suh, *Pseudo-Einstein real hypersurfaces in complex hyperbolic two-plane Grassmannians*, Advances in Appl. Math., **57**(2016), in press.
- Y.J. Suh, *Pseudo-Einstein real hypersurfaces in the complex quadric*, submitted.
- Y.J. Suh, *Pseudo-anti commuting and Ricci soliton real hypersurfaces in the complex quadric*, submitted.

References V

- Y.J. Suh, *Real hypersurfaces in the complex quadric with harmonic curvature*, Journal de Math. Pures Appl., **103**(2016), in press.
- Y.J. Suh, *Pseudo-Einstein real hypersurfaces in complex hyperbolic two-plane Grassmannians*, Advances in Appl. Math., **57**(2016), in press.
- Y.J. Suh, *Pseudo-Einstein real hypersurfaces in the complex quadric*, submitted.
- Y.J. Suh, *Pseudo-anti commuting and Ricci soliton real hypersurfaces in the complex quadric*, submitted.

References V

- Y.J. Suh, *Real hypersurfaces in the complex quadric with harmonic curvature*, Journal de Math. Pures Appl., **103**(2016), in press.
- Y.J. Suh, *Pseudo-Einstein real hypersurfaces in complex hyperbolic two-plane Grassmannians*, Advances in Appl. Math., **57**(2016), in press.
- Y.J. Suh, *Pseudo-Einstein real hypersurfaces in the complex quadric*, submitted.
- Y.J. Suh, *Pseudo-anti commuting and Ricci soliton real hypersurfaces in the complex quadric*, submitted.

References VI

- J. Berndt and Y.J. Suh, *Real hypersurfaces with isometric Reeb flow in complex two-plane Grassmannians*, Monatschefte fur Math. **137**(2002), 87-98.
- J. Berndt and Y.J. Suh, *Real hypersurfaces in the noncompact Grassmannians* $SU_{2,m}/S(U_2{\cdot}U_m)$, International J. of Math., **23**(2012), 1250103(35 pages).
- J. Berndt and Y.J. Suh, *Real hypersurfaces with isometric Reeb flow in complex quadrics*, International J. Math., **24**(2013), 1350050(18 pages).
- J. Berndt and Y.J. Suh, *Contact Real hypersurfaces in complex quadrics*, Proc. Amer. Math. Soc. **23**(2015), 2637-2649.

References VI

- J. Berndt and Y.J. Suh, *Real hypersurfaces with isometric Reeb flow in complex two-plane Grassmannians*, Monatschefte fur Math. **137**(2002), 87-98.
- J. Berndt and Y.J. Suh, *Real hypersurfaces in the noncompact Grassmannians* $SU_{2,m}/S(U_2{\cdot}U_m)$, International J. of Math., **23**(2012), 1250103(35 pages).
- J. Berndt and Y.J. Suh, *Real hypersurfaces with isometric Reeb flow in complex quadrics*, International J. Math., **24**(2013), 1350050(18 pages).
- J. Berndt and Y.J. Suh, *Contact Real hypersurfaces in complex quadrics*, Proc. Amer. Math. Soc. **23**(2015), 2637-2649.

References VI

- J. Berndt and Y.J. Suh, *Real hypersurfaces with isometric Reeb flow in complex two-plane Grassmannians*, Monatschefte fur Math. **137**(2002), 87-98.
- J. Berndt and Y.J. Suh, *Real hypersurfaces in the noncompact Grassmannians* $SU_{2,m}/S(U_2{\cdot}U_m)$, International J. of Math., **23**(2012), 1250103(35 pages).
- J. Berndt and Y.J. Suh, *Real hypersurfaces with isometric Reeb flow in complex quadrics*, International J. Math., **24**(2013), 1350050(18 pages).
- J. Berndt and Y.J. Suh, *Contact Real hypersurfaces in complex quadrics*, Proc. Amer. Math. Soc. **23**(2015), 2637-2649.

References VI

- J. Berndt and Y.J. Suh, *Real hypersurfaces with isometric Reeb flow in complex two-plane Grassmannians*, Monatschefte fur Math. **137**(2002), 87-98.
- J. Berndt and Y.J. Suh, *Real hypersurfaces in the noncompact Grassmannians* $SU_{2,m}/S(U_2{\cdot}U_m)$, International J. of Math., **23**(2012), 1250103(35 pages).
- J. Berndt and Y.J. Suh, *Real hypersurfaces with isometric Reeb flow in complex quadrics*, International J. Math., **24**(2013), 1350050(18 pages).
- J. Berndt and Y.J. Suh, *Contact Real hypersurfaces in complex quadrics*, Proc. Amer. Math. Soc. **23**(2015), 2637-2649.

References VII

- P.B. Eberlein, *Geometry of Non positively Curved Manifolds*, Chicago Lectures in Math., The Univ. of Chicago Press, **1996**,
- A.W. Knapp, *Lie Groups beyond an Introduction*, Progress in Math., Birkhäuser, **2002**,
- S. Helgason, *Differential Geometry, Lie Group and Symmetric Spaces*, Graduate Studies in Mathematics **34**, Amer. Math. Soc. **2001**,
- S. Helgason, *Geometric Analysis on Symmetric Spaces*, The 2nd Edition, Math. Survey and Monographs **39**, Amer. Math. Soc. **2008**.

References VII

- P.B. Eberlein, *Geometry of Non positively Curved Manifolds*, Chicago Lectures in Math., The Univ. of Chicago Press, **1996**,
- A.W. Knapp, *Lie Groups beyond an Introduction*, Progress in Math., Birkhäuser, **2002**,
- S. Helgason, *Differential Geometry, Lie Group and Symmetric Spaces*, Graduate Studies in Mathematics **34**, Amer. Math. Soc. **2001**,
- S. Helgason, *Geometric Analysis on Symmetric Spaces*, The 2nd Edition, Math. Survey and Monographs **39**, Amer. Math. Soc. **2008**.

References VII

- P.B. Eberlein, *Geometry of Non positively Curved Manifolds*, Chicago Lectures in Math., The Univ. of Chicago Press, **1996**,
- A.W. Knapp, *Lie Groups beyond an Introduction*, Progress in Math., Birkhäuser, **2002**,
- S. Helgason, *Differential Geometry, Lie Group and Symmetric Spaces*, Graduate Studies in Mathematics **34**, Amer. Math. Soc. **2001**,
- S. Helgason, *Geometric Analysis on Symmetric Spaces*, The 2nd Edition, Math. Survey and Monographs **39**, Amer. Math. Soc. **2008**.

References VII

- P.B. Eberlein, *Geometry of Non positively Curved Manifolds*, Chicago Lectures in Math., The Univ. of Chicago Press, **1996**,
- A.W. Knapp, *Lie Groups beyond an Introduction*, Progress in Math., Birkhäuser, **2002**,
- S. Helgason, *Differential Geometry, Lie Group and Symmetric Spaces*, Graduate Studies in Mathematics **34**, Amer. Math. Soc. **2001**,
- S. Helgason, *Geometric Analysis on Symmetric Spaces*, The 2nd Edition, Math. Survey and Monographs **39**, Amer. Math. Soc. **2008**.

ENDING

THANKS FOR *YOUR ATTENTION!*