Exercise 1: Solution proposed by José Carlos Martínez (2016-17 student)

You need to design a microstipy line for a high speed digital signal in a 4 layer PCB. The copper foil thickness of all planes is 35 um. Imagine you have your signal on the bottom layer, which should be the width of the 5.6 GHz signal strap to obtain a 50 Ohms impedance using Wheeler (Gupta, Garg, & Bahl, 1979), Hammerstad (Edwards & Steer, 2016), Ownes (Edwards & Steer, 2016; Kirschning, Jansen, & Koster, 1981), Hammerstad & Jensen (Edwards & Steer, 2016; Kirschning et al., 1981; Wong, 1979), Bahl & Garg [(Wong, 1979) y Kobayashi (Garg & Bahl, 1979)

Notación:

H = Substrate Thickness

W = Strip Width

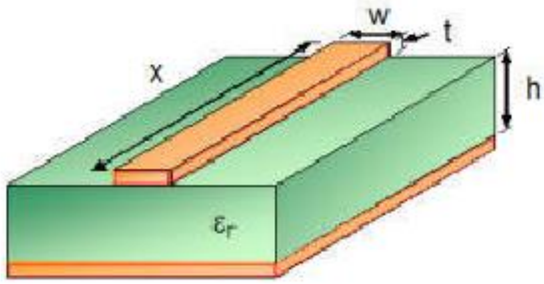
f = Frequency (GHz)

ε_r = Relative Permittivity

ε_{eff} = Effective Permittivity

C = Strip Capacitance

Z_0 = Characteristic Impedance (Ω)



Initial Values:

$H := 60mil$

$Z_0 := 50\Omega$

$\varepsilon_r := 4.3$

$f := 5.6GHz$

Wheeler

The expression proposed by Wheeler can be found at:
http://ieeexplore.ieee.org/stamp/stamp.jsp?arnumber=1129179
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$$w'/h = 8\,\frac{\sqrt{\left[\exp\left(\frac{R}{42.4}\sqrt{k+1}\right)-1\right]\frac{7+4/k}{11}+\frac{1+1/k}{0.81}}}{\left[\exp\left(\frac{R}{42.4}\sqrt{k+1}\right)-1\right]} \qquad (9)$$

$$R = \frac{42.4}{\sqrt{k+1}}\ln\left\{1+\left(\frac{4h}{w'}\right)\left[\left(\frac{14+8/k}{11}\right)\left(\frac{4h}{w'}\right) + \sqrt{\left(\frac{14+8/k}{11}\right)^2\left(\frac{4h}{w'}\right)^2+\frac{1+1/k}{2}\pi^2}\right]\right\}. \qquad (10)$$

$$Z_0 := \frac{Z_0}{1\Omega}$$

The denominator of the equation 9 is been set as A to simplify the expression.

$$\mathbf{A} = 1\,\mathbf{A}$$

$$A := e^{\left(\frac{Z_0}{42.4}\cdot\sqrt{\varepsilon_r+1}\right)} - 1$$

$$W := H\cdot\frac{8\cdot\left(\frac{7\varepsilon_r+4}{11\varepsilon_r}\cdot A+\frac{\varepsilon_r+1}{0.81\varepsilon_r}\right)^{\frac{1}{2}}}{A}$$

$$C := \frac{\sqrt{\varepsilon_r}}{c\cdot Z_0\cdot\Omega}$$

So the results using Wheeler are:

$$W = 2.956\cdot mm$$

$$C = 138.339\,\frac{1}{m}\cdot pF$$

$$W\_Wheeler := W$$

Ownes

Once again, the expression proposed by Ownes can be found at IEEE repository:
http://ieeexplore.ieee.org/stamp/stamp.jsp?tp=&arnumber=5269067
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$$\frac{h}{W} = \frac{\exp(H')}{8} - \frac{1}{4\exp(H')}$$

3.1 Calculation of W/h

For narrow strips, equations (3), (4) and (5) are used, i.e.

$$W/h = \left[\frac{\exp H'}{8} - \frac{1}{4\exp H'}\right]^{-1} \qquad (3A)$$

where

$$H' = \frac{Z_0\sqrt{2(\varepsilon_r+1)}}{119{\cdot}9} + \tfrac{1}{2}\left(\frac{\varepsilon_r-1}{\varepsilon_r+1}\right)\left(\ln\frac{\pi}{2} + \frac{1}{\varepsilon_r}\ln\frac{4}{\pi}\right) \qquad (4A)$$

For wide strips, Wheeler gives the equation:

$$W/h = 2/\pi[(d_\varepsilon-1)-\ln(2d_\varepsilon-1)] + \frac{\varepsilon_r-1}{\pi\varepsilon_r}\left[\ln(d_\varepsilon-1)+0{\cdot}293-\frac{0{\cdot}517}{\varepsilon_r}\right] \qquad (13)$$

$$\lambda_0 := \frac{c}{f}$$

$$\lambda_0 = 0.054\,\mathrm{m}$$

Again , for the sake of simplicity, we set a new variable, A:

$$A := Z_0\cdot\frac{\sqrt{2(\varepsilon_r+1)}}{119.9} + \frac{\varepsilon_r-1}{2(\varepsilon_r+1)}\cdot\left(0.4516 + \frac{0.2416}{\varepsilon_r}\right)$$

$$B := \frac{59.96\cdot\pi^2}{Z_0\cdot\sqrt{\varepsilon_r}}$$

Now , we analyse together the equations for narrow and wide strips.

$$U := \begin{cases} \dfrac{8}{e^{A} - 2\cdot e^{-A}} & \text{if } Z_0 > (44 - 2\varepsilon_r) \\ \dfrac{2}{\pi}\cdot\left[B - 1 - \ln(2\cdot B - 1) + \dfrac{\varepsilon_r - 1}{2\cdot\varepsilon_r}\cdot\left(\ln(B-1) + 0.293 - \dfrac{0.517}{\varepsilon_r}\right)\right] & \text{otherwise} \end{cases}$$

pared with the C270 results are shown in Fig. 5. The optimum changeover point between equations is this time more dependent on ε_r. A good compromise between small discontinuity between curves and low absolute error is achieved if the changeover is made when $Z_0 = (44-2\varepsilon_r)$ ohms. The discontinuity is then about

As for the effective permittiviy, in the same source we found, for narrow and wide strips, respectively:

$$H' = \ln\left[\frac{4h}{W} + \sqrt{16\left(\frac{h}{W}\right)^2 + 2}\right] \qquad (6)$$

$$\varepsilon_{e0} = \frac{\varepsilon_r+1}{2}\left[1 - \frac{1}{2H'}\left(\frac{\varepsilon_r-1}{\varepsilon_r+1}\right)\left(\ln\frac{\pi}{2} + \frac{1}{\varepsilon_r}\ln\frac{4}{\pi}\right)\right]^{-2} \qquad (9)$$

$$\varepsilon_{e0} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2}\left(1 + 10\frac{h}{W}\right)^{-0.5} \qquad (11)$$

$W := U \cdot H$

$$H_1 := \ln\left[\frac{4H}{W} + \left[\left(\frac{4H}{W}\right)^2 + 2\right]^{\frac{1}{2}}\right]$$

$$\varepsilon_{eff} := \begin{cases} \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2} \cdot \left(1 + \frac{10H}{W}\right)^{-0.555} & \text{if } \frac{W}{H} > 1 \\ \frac{\varepsilon_r + 1}{2} \cdot \left[1 - \frac{\varepsilon_r - 1}{2H_1 \cdot (\varepsilon_r + 1)} \cdot \left(0.4516 + \frac{0.2416}{\varepsilon_r}\right)\right]^{-2} & \text{otherwise} \end{cases}$$

It yields to an effective wavelength of:

$$\lambda_{eff} := \frac{\lambda_0}{\sqrt{\varepsilon_{eff}}}$$

So the results using the equations by Owens is:

$W = 2.964 \cdot mm$

$\lambda_{eff} = 29.684 \cdot mm$

$\varepsilon_{eff} = 3.253$

And the wavelength speed in this material:

$v_{eff} := f \cdot \lambda_{eff}$

$v_{eff} = 1.662 \times 10^8 \, m \cdot s^{-1}$

$W\_Ownes := W$

$\varepsilon_{eff\_Ownes} := \varepsilon_{eff}$

$\lambda_{eff\_Ownes} := \lambda_{eff}$

Hammerstand

http://ieeexplore.ieee.org/stamp/stamp.jsp?tp=&arnumber=4130821

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$$w/h = \begin{cases} 8/\{\exp(A) - 2\exp(-A)\} & , w/h \le 2 \\ \frac{2}{\pi}\left\{B - 1 - \ln(2B-1) + \frac{\varepsilon_r - 1}{2\varepsilon_r}\{\ln(B-1) + 0.39 - 0.61/\varepsilon_r\}\right\} & , w/h \ge 2 \end{cases} \qquad (6)$$

where

$$A = \frac{\pi}{\eta_0}\sqrt{2(\varepsilon_r + 1)}\,Z + \frac{\varepsilon_r - 1}{\varepsilon_r + 1}(0.23 + 0.11/\varepsilon_r) \qquad , B = \frac{\pi\eta_0}{2\sqrt{\varepsilon_r}\,Z}$$

$\lambda_0 := \frac{c}{f}$

$\lambda_0 = 0.054 \, m$

We follow the same pattern again:

$$A := \frac{Z_0}{60} \cdot \sqrt{\frac{\varepsilon_r + 1}{2}} + \frac{\varepsilon_r - 1}{\varepsilon_r + 1} \cdot \left(0.23 + \frac{0.11}{\varepsilon_r}\right)$$

$$B := \frac{377 \cdot \pi}{2 \cdot Z_0 \cdot \sqrt{\varepsilon_r}}$$

$$U := \begin{cases} \dfrac{8 \cdot e^{A}}{e^{2 \cdot A} - 2} & \text{if } \left(\dfrac{8 \cdot e^{A}}{e^{2 \cdot A} - 2}\right) < 2 \\ \dfrac{2}{\pi} \cdot \left[B - 1 - \ln(2 \cdot B - 1) + \dfrac{\varepsilon_r - 1}{2 \cdot \varepsilon_r} \cdot \left(\ln(B - 1) + 0.39 - \dfrac{0.61}{\varepsilon_r}\right)\right] & \text{otherwise} \end{cases}$$

Equations for effective permittivity:

$$\varepsilon_e = \frac{1}{2}\left\{\varepsilon_r + 1 + (\varepsilon_r - 1)\, F\right\}$$

Being F:

$$(1 + 12\ h/w)^{-1/2}$$

$$W := U \cdot H$$

$$\varepsilon_{eff} := \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2} \cdot \frac{1}{\sqrt{1 + 12 \cdot \frac{H}{W}}}$$

$$\lambda_{eff} := \frac{\lambda_0}{\sqrt{\varepsilon_{eff}}}$$

Finally, the results are:

$W = 2.964 \cdot mm$

$\lambda_{eff} = 29.622 \cdot mm$

$\varepsilon_{eff} = 3.266$

$v_{eff} := f \cdot \lambda_{eff}$

$v_{eff} = 1.659 \times 10^{8}\, m \cdot s^{-1}$

$W\_Hammerstad := W$

$\lambda_{eff\_Hammerstad} := \lambda_{eff}$

$\varepsilon_{eff\_Hammerstad} := \varepsilon_{eff}$

$W\_Wheeler = 2.956 \cdot mm$

$W\_Hammerstad = 2.964 \cdot mm$

$\varepsilon_{eff\_Hammerstad} = 3.266$

$\lambda_{eff\_Hammerstad} = 29.622 \cdot mm$

$W\_Ownes = 2.964 \cdot mm$

$\varepsilon_{eff\_Ownes} = 3.253$

$\lambda_{eff\_Ownes} = 29.684 \cdot mm$

Now , we will calculate the impedance of the strip assuming the width obtained before. Again, different methods will be used:

Hammerstand & Jensen

These equations do not take into account substrate thickness. However, the accuracy of this model is better than 2% de este modelo es mejor del 2% para ε_r< 128 and 0.01< W/H <100.

http://ieeexplore.ieee.org/stamp/stamp.jsp?arnumber=1124303
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$$\varepsilon_e(u,\varepsilon_r) = \frac{\varepsilon_r+1}{2} + \frac{\varepsilon_r-1}{2}\left(1+\frac{10}{u}\right)^{-a(u)\, b(\varepsilon_r)} \qquad (3)$$

$$a(u) = 1+\frac{1}{49}\ln\frac{u^4+(u/52)^2}{u^4+0.432} + \frac{1}{18.7}\ln\left[1+\left(\frac{u}{18.1}\right)^3\right] \qquad (4)$$

$$b(\varepsilon_r) = 0.564\left(\frac{\varepsilon_r-0.9}{\varepsilon_r+3}\right)^{0.053} \qquad (5)$$

For the width determined by Wheeler's equations:

$\mathbf{W} := \mathbf{W\_Wheeler}$

$$\mathbf{a} := 1+\frac{1}{49}\cdot\ln\left[\frac{\left(\frac{W}{H}\right)^4+\left(\frac{W}{52H}\right)^2}{\left(\frac{W}{H}\right)^4+0.432}\right] + \frac{1}{18.7}\cdot\ln\left[1+\left(\frac{W}{18.1H}\right)^3\right] = 0.999$$

$$\mathbf{b} := 0.564\cdot\left(\frac{\varepsilon_r-0.9}{\varepsilon_r+3}\right)^{0.053} = 0.542$$

$$\varepsilon_{eff} := \frac{\varepsilon_r+1}{2}+\frac{\varepsilon_r-1}{2}\cdot\left(1+\frac{10H}{W}\right)^{-a\cdot b}$$

Effective Permittivity:

$$Z_{01}(u) = \frac{\eta_o}{2\pi}\ln\left[\frac{f(u)}{u}+\sqrt{1+\left(\frac{2}{u}\right)^2}\right]$$

$$f(u) = 6+(2\pi-6)\exp\left[-\left(\frac{30.666}{u}\right)^{0.7528}\right]$$

$$F := 6+(2\pi-6)\cdot e^{-\left(\frac{30.666H}{W}\right)^{0.7528}}$$

$$Z_0 := \frac{60}{\sqrt{\varepsilon_{eff}}}\cdot\ln\left[\frac{F\cdot H}{W}+\sqrt{1+\left(\frac{2H}{W}\right)^2}\right]$$

It yields to:

$Z_0 = 50.15$

$\varepsilon_{eff} = 3.267$

For the width determined by Ownes' equations:

$\mathbf{W} := \mathbf{W\_Ownes}$

$$a := 1 + \frac{1}{49}\cdot\ln\left[\frac{\left(\frac{W}{H}\right)^4 + \left(\frac{W}{52H}\right)^2}{\left(\frac{W}{H}\right)^4 + 0.432}\right] + \frac{1}{18.7}\cdot\ln\left[1 + \left(\frac{W}{18.1H}\right)^3\right] = 0.999$$

$$b := 0.564\cdot\left(\frac{\varepsilon_r - 0.9}{\varepsilon_r + 3}\right)^{0.053} = 0.542$$

$$\varepsilon_{eff} := \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2}\cdot\left(1 + \frac{10H}{W}\right)^{-a\cdot b}$$

$$F := 6 + (2\pi - 6)\cdot e^{-\left(\frac{30.666H}{W}\right)^{0.7528}}$$

$$Z_0 := \frac{60}{\sqrt{\varepsilon_{eff}}}\cdot\ln\left[\frac{F\cdot H}{W} + \sqrt{1 + \left(\frac{2H}{W}\right)^2}\right]$$

$Z_0 = 50.069$

$\varepsilon_{eff} = 3.268$

For the width determined by Hammerstad's equations:

$W := W\_Hammerstad$

$$a := 1 + \frac{1}{49}\cdot\ln\left[\frac{\left(\frac{W}{H}\right)^4 + \left(\frac{W}{52H}\right)^2}{\left(\frac{W}{H}\right)^4 + 0.432}\right] + \frac{1}{18.7}\cdot\ln\left[1 + \left(\frac{W}{18.1H}\right)^3\right] = 0.999$$

$$b := 0.564\cdot\left(\frac{\varepsilon_r - 0.9}{\varepsilon_r + 3}\right)^{0.053} = 0.542$$

$$\varepsilon_{eff} := \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2}\cdot\left(1 + \frac{10H}{W}\right)^{-a\cdot b}$$

$$F := 6 + (2\pi - 6)\cdot e^{-\left(\frac{30.666H}{W}\right)^{0.7528}}$$

$$Z_0 := \frac{60}{\sqrt{\varepsilon_{eff}}}\cdot\ln\left[\frac{F\cdot H}{W} + \sqrt{1 + \left(\frac{2H}{W}\right)^2}\right]$$

So the results are:

$Z_0 = 50.066$

$\varepsilon_{eff} = 3.268$

According to Hammerstad&Jensen's method, the equations which provide the strip impedance nearest to 50 ohms is Hammerstad's.

Bahl & Garg and Kobayashi

Now, the process is repeated, using Bahl & Garg and Kobayashi method:

We start with Ownes' results. To take into account dispersion frequency Kobayashi equations are used:

http://ieeexplore.ieee.org/stamp/stamp.jsp?arnumber=3665

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$$\epsilon^*_{\text{eff}}(f) = \epsilon^* - \frac{\epsilon^* - \epsilon^*_{\text{eff}}(0)}{1 + (f/f_{50})^m} \qquad (9)$$

where

$$f_{50} = \frac{f_{K,\text{TM}_0}}{0.75 + \left(0.75 - \frac{0.332}{\epsilon^{*1.73}}\right)\frac{w}{h}} \qquad (10)$$

$$f_{K,\text{TM}_0} = \frac{c \tan^{-1}\left\{\epsilon^* \sqrt{\frac{\epsilon^*_{\text{eff}}(0) - 1}{\epsilon^* - \epsilon^*_{\text{eff}}(0)}}\right\}}{2\pi h \sqrt{\epsilon^* - \epsilon^*_{\text{eff}}(0)}} \qquad (11)$$

$$\mathbf{W} := \mathbf{W\_Ownes}$$

$$\varepsilon_{\text{eff}} := \varepsilon_{\text{eff\_Ownes}}$$

$$\mathbf{f_{TM0}} := \frac{\mathbf{c}}{2\pi \mathbf{H} \cdot \sqrt{\varepsilon_r - \varepsilon_{\text{eff}}}} \cdot \text{atan}\left(\varepsilon_r \cdot \sqrt{\frac{\varepsilon_{\text{eff}} - 1}{\varepsilon_r - \varepsilon_{\text{eff}}}}\right)$$

$$\mathbf{f_{50}} := \frac{\mathbf{f_{TM0}}}{0.75 + \left(0.75 - 0.332 \cdot \varepsilon_r^{-1.73}\right) \cdot \frac{\mathbf{W}}{\mathbf{H}}}$$

As for the dispersion frequency in this method we have the following expressions:

$$m = m_0 m_c \ (\leqslant 2.32) \qquad (12)$$

$$m_0 = 1 + \frac{1}{1 + \sqrt{w/h}} + 0.32\left(\frac{1}{1 + \sqrt{w/h}}\right)^3 \qquad (13)$$

$$m_c \begin{cases} = 1 + \frac{1.4}{1 + \frac{w}{h}}\left\{0.15 - 0.235 \exp\left(\frac{-0.45 f}{f_{50}}\right)\right\} & (w/h \leqslant 0.7) \qquad (14) \\ = 1 & (w/h \geqslant 0.7). \qquad (15) \end{cases}$$

$$\mathbf{m_0} := 1 + \frac{1}{1 + \sqrt{\frac{\mathbf{W}}{\mathbf{H}}}} + 0.32 \cdot \left(\frac{1}{1 + \sqrt{\frac{\mathbf{W}}{\mathbf{H}}}}\right)^3$$

$$\mathbf{m_c} := \begin{cases} 1 + \frac{1.4}{1 + \frac{\mathbf{W}}{\mathbf{H}}} \cdot \left(0.15 - 0.235 \cdot e^{\frac{-0.45 \cdot \mathbf{f}}{\mathbf{f_{50}}}}\right) & \textbf{if } \frac{\mathbf{W}}{\mathbf{H}} \le 0.7 \\ 1 & \textbf{otherwise} \end{cases}$$

$$\mathbf{m} := \begin{cases} \mathbf{m_0} \cdot \mathbf{m_c} & \textbf{if } \mathbf{m_0} \cdot \mathbf{m_c} \le 2.32 \\ 2.32 & \textbf{otherwise} \end{cases}$$

$$\varepsilon_{\text{eff\_F}} := \varepsilon_r - \frac{\varepsilon_r - \varepsilon_{\text{eff}}}{1 + \left(\frac{\mathbf{f}}{\mathbf{f_{50}}}\right)^{\mathbf{m}}}$$

$$Z_{0\_F} := Z_0 \cdot \frac{\varepsilon_{eff\_F} - 1}{\varepsilon_{eff} - 1} \cdot \sqrt{\frac{\varepsilon_{eff}}{\varepsilon_{eff\_F}}}$$

In this case, the results are:

$Z_{0\_F} = 52.123$

$\varepsilon_{eff\_F} = 3.396$

$Z_{0\_Kobayashi\_WOwnes.} := Z_{0\_F}$

$\varepsilon_{eff\_Kobayashi\_WOwnes} := \varepsilon_{eff\_F}$

<u>For the width determined by Hammerstad's equations:</u>

$W := W\_Hammerstad$

$\varepsilon_{eff} := \varepsilon_{eff\_Hammerstad}$

$$f_{TM0} := \frac{c}{2\pi H \cdot \sqrt{\varepsilon_r - \varepsilon_{eff}}} \cdot \text{atan}\left(\varepsilon_r \cdot \sqrt{\frac{\varepsilon_{eff} - 1}{\varepsilon_r - \varepsilon_{eff}}}\right)$$

$$f_{50} := \frac{f_{TM0}}{0.75 + \left(0.75 - 0.332 \cdot \varepsilon_r^{-1.73}\right) \cdot \frac{W}{H}}$$

$$m_0 := 1 + \frac{1}{1 + \sqrt{\frac{W}{H}}} + 0.32 \cdot \left(\frac{1}{1 + \sqrt{\frac{W}{H}}}\right)^3$$

$$m_c := \begin{cases} 1 + \frac{1.4}{1 + \frac{W}{H}} \cdot \left(0.15 - 0.235 \cdot e^{\frac{-0.45 \cdot f}{f_{50}}}\right) & \text{if } \frac{W}{H} \le 0.7 \\ 1 & \text{otherwise} \end{cases}$$

$$m := \begin{cases} m_0 \cdot m_c & \text{if } m_0 \cdot m_c \le 2.32 \\ 2.32 & \text{otherwise} \end{cases}$$

$$\varepsilon_{eff\_F} := \varepsilon_r - \frac{\varepsilon_r - \varepsilon_{eff}}{1 + \left(\frac{f}{f_{50}}\right)^m}$$

$$Z_{0\_F} := Z_0 \cdot \frac{\varepsilon_{eff\_F} - 1}{\varepsilon_{eff} - 1} \cdot \sqrt{\frac{\varepsilon_{eff}}{\varepsilon_{eff\_F}}}$$

Finally, the results:

$Z_{0\_F} = 52.064$

$\varepsilon_{eff\_F} = 3.407$

$Z_{0\_Kobayashi\_WHammerstad} := Z_{0\_F}$

$\varepsilon_{eff\_Kobayashi\_WHammerstad} := \varepsilon_{eff\_F}$

According to **Bahl & Garg and Kobayashi** method, the equations which provide the strip impedance nearest to 50 ohms is Hammerstad's.

$Z_{0\_Kobayashi\_WOwnes.} = 52.123$

$\varepsilon_{eff\_Kobayashi\_WOwnes} = 3.396$

$Z_{0\_Kobayashi\_WHammerstad} = 52.064$

$\epsilon_{eff\_Kobayashi\_WHammerstad} = 3.407$
