

2023 Mini-course

Title: Fractional GJMS operators and applications

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Abstract:

In a paper of R. Graham and M. Zworski in 2003, they have re-introduced the GJMS operators on the boundary of conformally compact Einstein (CCE) manifolds via the study of scattering matrix. In this mini-course, I will further expose this connection and present some geometric applications.

In the first two lectures, I will start with a brief description of background and known results –mainly the earlier works of R. Graham and J. Lee – and the study of Poisson equations on CCE manifolds and the scattering matrix of order 2γ . I will describe a parameter of family of the Poisson equations (based on some earlier joint works with M. Gonzalez and J. Case and myself), with the goal of studying the scattering matrix when γ is a fractional number $\gamma \in (0, \frac{n}{2})$, where n is the dimension of the boundary of the CCE manifold, we then define a parameter of "adapted metrics".

As geometric applications, we will derive a comparison result between the boundary Yamabe constant and the interior Yamabe constants on CCE manifolds; and describe some Liouville type result of the Poisson equation on the hyperbolic space in the setting of the upper half space R_+^{n+1} .

In the third lecture, we study the problem of "conformal filling in". Namely, given a manifold $(M^n; [h])$, when is it the boundary of a conformally compact Einstein manifold $(X^{n+1}; g_+)$ with $r^2g_+|_M = h$ for some defining function r on X^{n+1} ?

I will briefly mention some more recent progress made regarding the "existence" and "uniqueness" of above problem; then report some joint works with Yuxin Ge, in which we study the "compactness" issue of the problem.