Convergence to equilibrium for the linear Becker-Döring equations

The Becker-Döring equations, introduced by Becker and Döring (1935), are a classical model for coagulation and fragmentation processes, describing how clusters of particles grow or break apart over time. They arise in a wide range of applications, including phase transitions, condensation phenomena, and aggregation processes in biological systems.

While significant research has focused on the nonlinear case, including the study of metastable states by Penrose (1989) and the exponential convergence results in the subcritical case by Cañizo Lods (2013), in this talk, we focus on the linear Becker-Döring equations, corresponding to the case where the concentration of monomers is kept constant. In this setting, the evolution of the cluster densities $c_i(t)$ is governed by

$$\frac{d}{dt}c_i(t) = W_{i-1} - W_i, \quad \text{for all } i \geq 2,$$

where the fluxes W_i describe the balance between coagulation and fragmentation mechanisms.

Although the total mass of the system is not conserved in this linear case, the equations admit explicit equilibrium solutions. We study the long-time behavior of solutions and show that they converge exponentially fast towards equilibrium under natural assumptions on the model coefficients. The analysis combines ideas from entropy methods (Kreer (1993)) and spectral theory, leading to quantitative estimates of the convergence rate in both subcritical and supercritical regimes.