Gradient flows face-to-face 5 in Graná

Granada, September $16^{\text{th}} - 19^{\text{th}}$, 2025

https://wpd.ugr.es/~imag/events/event/gradient-flows/

Program

Day	Tuesday 16	Wednesday 17	Thursday 18	Friday 19
09:30–10:15	Matthes	Zimmer	Hoffmann	Cancès
10:15–11:00	Yoldaş	Gárriz	Zhu	Skrzeczkowski
11:00–11:30	**Coffee Break**			
11:30–12:15	Fagioli	Wu	Heinze	Simonov
12:15–13:00	Herda	Tse	Warren	**Closing**
13:30–16:00	**Lunch**			
16:00–17:00	Short talks 1	Short talks 2	Short talks 3	
17:00–18:30	**Discussion time**			

The workshop will take place at IMAG (location) on Tuesday, Thursday and Friday. As an exception, on Wednesday the workshop will take place at Carmen de la Victoria (location).

Short talks (16:00-17:00)

(Each talk is 10 minutes long.)

Tuesday session

- Inma Benitez (University of Granada):
  Asymptotic behavior of solutions to the Becker-Döring equations.
- Sofiane Cherf (Université Claude Bernard - Lyon 1):
  A Bound between the JKO and Entropic JKO Schemes.
- Alejandro Fernandez-Jimenez (University of Oxford):
  A Li–Yau and Aronson Bénilam approach to study Keller–Segel.
- Fatemeh Ghaderi Zefreh (University of L'Aquila):
  Kermack-McKendrick type models for epidemics with nonlocal aggregation terms.
- Fanch Coudreuse (Université Claude Bernard - Lyon 1):
  Li-Yau-Hamilton type inequalities for the JKO scheme on the torus.
- Julián Cabrera-Nyst (University of Granada):
  Mean field limit of non exchangeable interacting diffusions on co-evolutionary networks

Wednesday session (until 17:10)

- Nicolai Gerber (University of Ulm):
  Formation of clusters and coarsening in weakly interacting diffusions.
- Shuchen Guo (University of Oxford):
  Mean-Field Derivation of the Space-Homogeneous Landau Equation via BBGKY Hierarchy Method.
- Daniel Morris (TU Delft):
  Nonlinear diffusions as limits of kinetic equations.
- Mary Chriselda Antony Oliver (University of Cambdrige):
  Consistency of Gradient Flows for Laplace Learning.
- Guy Parker (Durham University):
  On a Cross-Diffusion System with Independent Drifts and no Self-Diffusion: The Existence of Totally Mixed Solutions.

- Megha Kamath Katapady (University of L'Aquila):
  Bulk-surface Cahn Hilliard model for chemically active wetting

- Niccolò Tassi (University of Granada):
  Time Asymptotics and Diffusive Limits in L^p for some Linear Kinetic Equations.

Thursday session

- Oscar de Wit (University of Cambridge):
  Collective behaviour of active particles with mean-field interaction.

- Juliane Krautz (Universität Augsburg):
  The dynamic Schrödinger problem on metric graphs.

- Kexin Lin (Université Claude Bernard - Lyon 1):
  Existence of the TV Wasserstein Gradient Flow via the JKO Scheme.

- Zhenyu Huang (University of Oxford):
  Mean field error estimate of the random batch method for large interacting particle system.

- Jethro Warnett (University of Oxford):
  Stein Variational Gradient Descent: From non-local to local.

- Holger Spellmann (University of Ulm):
  Preserving gradient system structures for multiscale chemical reaction networks

List of Abstracts

Discretizing the Fokker-Planck equation with second-order accuracy: a dissipation driven approach

Clément Cancès

Inria University of Lille, France

We propose a fully discrete finite volume scheme for the standard Fokker-Planck equation. The space discretization relies on the well-known square-root approximation, which falls into the framework of two-point flux approximations. Our time discretization is novel and relies on a tailored nonlinear mid-point rule, designed to accurately capture the dissipative structure of the model. We establish well-posedness for the scheme, positivity of the solutions, as well as a fully discrete energy-dissipation inequality mimicking the continuous one. We then prove the rigorous convergence of the scheme under mildly restrictive conditions on the unstructured grids, which can be easily satisfied in practice. Numerical simulations show that our scheme is second order accurate both in time and space, and that one can solve the discrete nonlinear systems arising at each time step using Newton's method with low computational cost.

On a chemotaxis model with nonlinear diffusion modelling multiple sclerosis

Simone Fagioli

University of L'Aquila, Italy

We investigated existence of global weak solutions for a system of chemotaxis type with nonlinear degenerate diffusion, arising in modelling Multiple Sclerosis disease. The model consists of three equations describing the evolution of macrophages (m), cytokine (c) and apoptotic oligodendrocytes (d). The main novelty in our work is the presence of a nonlinear diffusivity D(m), which may results to be more appropriate from the modelling point of view. We first show the existence of global bounded solutions for the model. We then investigate some properties on the stationary states and pattern formation studying a reduced one species problem.

The heat equation in the hyperbolic space: asymptotic behaviour

Alejandro Gárriz

University of Granada, Spain

We will present some new results about the large-times behaviour of solutions of the heat equation posed in the hyperbolic space $\mathbb{H}^d$. We will not only cover several different families of solutions presenting symmetries in their initial data but will also pay special attention to the general case of integrable initial data, improving the results of J.L. Vázquez in [1] and answering the questions therein. For such initial data the asymptotic profiles were not even known. We will manage to provide not only the asymptotic profiles but also the speed of convergence of the solutions thanks to the definition and the study of an Entropy natural to the problem. This is a joint work with J.A. Cañizo and D.A. Marín.

[1] Vázquez, J.L. Asymptotic behaviour for the heat equation in hyperbolic space. Communications in Analysis and Geometry (2018).

Two examples of gradient flows in continuity equation format

Georg Heinze

WIAS Berlin, Germany

In this talk we will discuss two (generalized) gradient flow models that benefit from writing them in the continuity equation formulation introduced by Peletier and Schlichting in 2023.

First, we consider drift-diffusion along the edges of a metric graph coupled to reservoirs at its vertices. After rewriting the coupled PDE-ODE system as a gradient flow, we use the recently developed framework of EDP convergence with embedding to establish well-posedness via a discretization of finite-volume type. Furthermore, we show convergence of rescaled systems to gradient flows on reduced metric or combinatorial graphs.

Secondly, we investigate reaction-diffusion systems on the d-dimensional torus. We construct appropriate space-discrete processes and show their convergence by again using the notion of EDP convergence with embedding. As an important step, we prove a chain rule inequality for the limit reaction-diffusion system, which features a non-convex dissipation functional.

This talk is based on joint works with Alexander Mielke, Jan-Frederik Pietschmann, André Schlichting and Artur Stephan.

Finite volumes for the local sensing chemotaxis system

Maxime Herda
Inria University of Lille, France

In this talk, I will present a finite volume scheme for a parabolic system which models chemotaxis with local sensing. This system has a similar gradient flow structure as the celebrated Keller-Segel system, but unlike the latter, its solutions are known to exist globally in 2D. The long-time behavior of solutions is only partially understood which motivates numerical exploration with reliable numerical methods. I will present the design of the scheme, its numerical analysis (convergence, asymptotic preserving properties...) and simulations. This is a work in collaboration with Ariane Trescases (Univ. Toulouse) and Antoine Zurek (UT Compiègne).

Monotone Multispecies Flows

Franca Hoffmann
Caltech, US

We introduce a notion of monotonicity for multi-species systems of partial differential equations governed by mass-preserving flow dynamics, extending monotonicity in Banach spaces to the Wasserstein-2 metric space. We show that monotonicity implies the existence of and convergence to a unique steady state, convergence of the velocity fields and second moments, and contraction in the Wasserstein-2 metric. In the special setting where each species follows a Wasserstein-2 gradient flow of its own energy, we prove convergence to the unique Nash equilibrium of the associated energies and delineate the relationship between monotonicity and displacement convexity. This extends known zero-sum results in infinite-dimensional game theory to the general-sum setting. Examples of such multi-species flows include cross-diffusion, gradient flows with potentials, nonlocal interaction, linear and non-linear diffusion, and min-max systems, and we also draw connections to a class of mean-field games.

Diffusion: not just discretized, but also quantized

Daniel Matthes
TU Munich, Germany

Around 2005, Degond, Mehats and Ringhofer proposed a novel approach for the derivation of quantum fluid models (known as QHD, QET etc) from first principles: they apply a moment method to the quantum Boltzmann equation, using a BGK collision operator for moment closure. In the simplest case, the resulting fluid model is the non-local quantum drift diffusion equation nlQDD. It turns out that this equation is formally the Wasserstein gradient flow of the relative von Neumann entropy.

In our (so far unsuccessful) attempt to understand the appearance of nlQDD's gradient flow structure, we have re-done the derivation by moment closure consistently on the level of spatial discretization, and we are able to replicate (albeit still not understand) the variational form in the discrete setting, along with essentially all the relevant estimates. We can further pass to the continuous limit, thereby generalizing en passant the so far only result on existence of weak solutions from close-to-equilibrium to large data, up to the first time at which the continuous solution generates vacuum.

This is mainly joint work with Eva-Maria Rott; the DLSS part is a joint work with Andre Schlichting and Giuseppe Savare.

The Logarithmic Sobolev Inequality: Stability, Instability, and Improved Convergence Rates for the Ornstein–Uhlenbeck Flow

Nikita Simonov

Sorbonne Université, France

In certain functional inequalities, the best constants and minimizers are known. The next natural question concerns stability: if a function "almost attains equality," in what sense is it close to a minimizer? We will discuss some recent results (both positive and negative) on quantitative stability for the Logarithmic Sobolev inequality. Our approach is based on a variant of the carré du champ method for the Ornstein–Uhlenbeck equation, enabling us to derive fully constructive estimates. This talk is based on a series of results obtained in collaboration with Giovanni Brigati (ISTA) and Jean Dolbeault (CEREMADE–Dauphine).

Optimal rate of convergence for a nonlocal-to-local limit in one dimension

Jakub Skrzeczkowski

University of Oxford, UK

We consider a nonlocal approximation of the quadratic porous medium equation where the pressure is given by a convolution with a mollification kernel. It is known that when the kernel concentrates around the origin, the nonlocal equation converges to the local one. However, the question of how fast it converges is not well-explored in the literature. In one spatial dimension, for a particular choice of the kernel, and under mere assumptions on the initial condition, we quantify the rate of convergence in the 2-Wasserstein distance. First, we reprove the recent result of Amassad and Zhou (2025) yielding the rate of $\sqrt{\varepsilon}$ using a simpler technique based on the Evolutionary Variational Inequality for both nonlocal and local equations. Next, using numerical simulations, we observe that the rate of $\sqrt{\varepsilon}$ is not optimal. Therefore, we obtain a new formula for the Wasserstein distance between two abstract gradient flows which, when applied to the problem, together with Aronson-Benilian estimates, yields the rate of ε, suggested to be optimal by numerics. This is joint work with J.A. Carrillo, S. Fronzoni (Oxford), C. Elbar (Lyon), P. Gwiazda (Warsaw).

Evolution of Gaussians in the Hellinger-Kantorovich-Boltzmann gradient flow

Oliver Tse

Eindhoven University of Technology (TU/e), The Netherlands

In this talk, I will share how gradient flows driven by the Boltzmann entropy in the Hellinger-Kantorovich geometry can induce gradient structures within a restricted class of Gaussian distributions. I will also discuss how the system converges to equilibrium, the type of convexity properties it exhibits, and how the decay rates can be sharpened by estimating the evolution of the covariance. If time permits, I will briefly address how some of these ideas extend to non-Gaussian settings.

Gradient flow structure for some nonlocal diffusion equations

Andrew Warren

University of British Columbia, Canada

We provide a “nonlocal version” of the result of Jordan, Kinderlehrer, and Otto, that the Fokker-Planck equation is a gradient flow of the relative entropy. Namely, we show that a class of ”nonlocal diffusion equations” including the anisotropic fractional heat equation can be cast as gradient flows of the relative entropy, with respect to a suitable ”nonlocal Wasserstein metric”. The result is more delicate than it may seem at first glance: for instance, in the heavy-tailed case, it is not known whether the minimizing movement scheme converges. An example of a corollary of this gradient flow structure is that we establish evolutionary Gamma-convergence with respect to perturbations of the nonlocality kernel and stationary measure.

Mean Field Limit for Congestion Dynamics in One Dimension

Jeremy Wu

University of Manitoba, Canada

In this talk, I will present joint work with Inwon Kim and Antoine Mellet in which we derive a model for congested transport (a PDE at a macroscopic scale) from particle dynamics (a system of ODEs at the microscopic scale). Such PDEs appear very naturally in the description of crowd motion, tumour growth, and general aggregation phenomena. We begin with a system where the particle trajectories evolve according to a gradient flow with a non-overlapping constraint; the particles have a fixed finite distance of separation from each other. This constraint leads to a Lagrange multiplier which, in the mean field limit (infinite number of particles), generates a pressure variable to enforce the hard-congestion constraint. We rely on both the Eulerian and Lagrangian perspectives for the continuum limit.

Macroscopic effects of an anisotropic Gaussian-type repulsive potential on a dense system of elongated particles

Havva Yoldaş

Delft University of Technology, The Netherlands

Elongated particles in dense systems often exhibit alignment due to volume exclusion interactions, leading to packing configurations. Traditional models of collective dynamics typically impose this alignment phenomenologically, neglecting the influence of volume exclusion on particle positions. I will talk about a recent work where we derive nematic alignment from an anisotropic repulsive potential, focusing on a Gaussian-type potential and first-order dynamics for the particles. We perform a hydrodynamic limit to uncover the effects of anisotropy on both particle density and direction. The talk is based a joint work with Sara Merino, Claudia Wytrzens both from University of Vienna and Steffen Plunder from University of Kyoto.

Two New Machine Learning Applications of Gradient Flows for Sampling

Jia-Jie Zhu

WIAS Berlin, Germany

This talk features two recent works on gradient flows for statistical sampling in machine learning. The first part of this talk focuses on the global exponential decay of the entropy functionals along the unbalanced-transport gradient flows, a.k.a. Hellinger-Kantorovich or Wasserstein-Fisher-Rao. As an example, we will construct a new formal dissipation geometry, which we term the interaction-force transport, that guarantees convergence of minimizing the maximum-mean discrepancy. This then results in a new algorithm for sampling and generative modeling. In the second part, I will introduce a gradient flow formulation to enhance the robustness of learning and optimization algorithms under data distribution shifts. Previously, researchers have interpreted these problems as interacting gradient flows. In comparison, our new formulation unifies the analysis and motivates the design of new and flexible algorithms. In particular, I will show using entropy-regularized optimal transport for learning robustly under distribution shifts.

PDE descriptions of finitely many interacting particles

Johannes Zimmer
TU Munich, Germany

We consider systems of finitely many interacting particles. The focus will be on models which are described by a second order Langevin equation, i.e., particles experiencing inertia, leading to Vlasov-Fokker-Planck type equations, but much of the analysis also applies to first order Langevin equations and related gradient flows. We introduce an associated equation of fluctuating hydrodynamics ("Dean-Kawasaki-type equation"), which is a stochastic PDE describing the collective motion. This equation can be interpreted as a stochastic version of a Vlasov-Fokker-Planck equation respectively of a gradient flow. We give a rigorous derivation of the PDE, and then show a dichotomy previously known for a class of purely diffusive systems: Solutions exist only for suitable atomic initial data, but not for smooth initial data. The class of systems covered includes several models of active matter. The talk will attempt to sketch the wider picture of mathematical aspects of fluctuating hydrodynamic, including implications for numerical methods for such equations.

Joint work with Fenna Müller and Max von Renesse.

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