Bifurcation in a Quasilinear Schrödinger-Type Equation with Two Parameters: Theory and Applications to Multiplicity of Solutions

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Abstract: In this talk we deal with the following family of equations,

$$\begin{cases} -\Delta u - \lambda m(x) u \Delta(u^2) = f(\mu, x, u) \text{ in } \Omega, \\ u = 0 \text{ on } \Omega. \end{cases} \qquad (P(\lambda, \mu))$$

where $\lambda, \mu \in \mathbb{R}$, Ω is an open and bounded subset of $\mathbb{R}^N$ with smooth boundary, $m(x)$ is a continuous function with $0 \leq m(x) \leq M$ and $f\colon \mathbb{R} \times \Omega \times \mathbb{R}^+ \to \mathbb{R}$ is a C^1 function such that $f(\mu, x, 0) = 0$ for all $x \in \Omega$ and $\mu \in \mathbb{R}$ and satisfies:

For every Γ bounded subset of $\mathbb{R} - \{0\}$ and $\mu \in \Gamma$,

$$\lim_{s \to 0^+} \frac{f(\mu, x, s)}{s} = \mu f'_+(x, 0), \quad \text{uniformly in } (\mu, x) \in \Gamma \times \Omega,$$

with $0 \leq f'_+(x, 0) \in L^\infty(\Omega)$ not identically zero.

With this conditions we prove the existence of a continuum of solutions, we study the laterality of the continuum and we give explicit examples in which the number of solutions changes depending on the parameter λ.

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