On the equidistribution of closed geodesics and geodesic nets

Given a Riemannian manifold (M^n, g), it is natural to ask whether it admits any closed geodesics and if so, how many are there and how are they distributed along the manifold. In this talk, I will present the following result: given a closed 2-manifold M^2, for a generic (in the Baire sense) Riemannian metric g on M^2 there exists an equidistributed sequence $\{\gamma_i\}_{i\in\mathbb{N}}$ of closed geodesics on (M^2, g). The same question in a higher dimensional ambient manifold M^n turns out to be much harder and is still widely open. However, we can approach the same problem but considering stationary geodesic nets (which are embedded graphs in (M, g) which are stationary with respect to the length functional) instead of closed geodesics. Together with Yevgeny Liokumovich, we showed that for a Baire-generic metric g on a fixed closed manifold M^n, the union of all embedded stationary geodesic nets on (M^n, g) is dense in M^n. I will also present the following stronger result: if $n \geq 3$, given a closed n-manifold M^n, for a Baire-generic metric g on M^n there exists an equidistributed sequence $\{\gamma_i\}_{i\in\mathbb{N}}$ of stationary geodesic nets on (M^n, g). For $n = 3$, this was joint work with Xinze Li.