

Título: Comparaciones tipo Talenti y desigualdades isoperimétricas en variedades Riemannianas.

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El establecimiento de resultados tipo Talenti para la simetrización de la solución de una ecuación de Poisson, planteada sobre un dominio regular D en una variedad Riemanniana (M, g) , se encuentra estrechamente vinculado a la existencia de una desigualdad isoperimétrica satisfecha por los dominios de la variedad. Esta relación ha sido explorada previamente en referencias como [Mc], [CLD], [DH] y [MV], tanto en el contexto compacto como en el no-compacto.

Hasta la fecha, el conocimiento sobre comparaciones en espacios de curvatura negativa es limitado, más allá de lo establecido por McDonald en [Mc] (donde se prueba una comparación tipo Talenti en espacios hiperbólicos). La dificultad radica en que, en espacios de curvatura negativa, no se ha establecido una desigualdad isoperimétrica de forma general.

Presentaremos un esquema de la demostración de una serie de comparaciones tipo Talenti para variedades Riemannianas completas y no compactas que cumplan una de las siguientes condiciones:

- Poseen una constante isoperimétrica positiva.
- Su perfil isoperimétrico está controlado en cierto sentido.

Estas hipótesis son integradoras en el sentido de que, en el caso no compacto, engloban los contextos estudiados en [Mc], [CLD] y [DH], e incluyen además a las variedades de Cartan-Hadamard.

Bibliografía

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[Mc] P. McDonald, Isoperimetric conditions, Poisson problems, and diffusions in Riemannian manifolds, *Potential Anal.* 16 (2), 115-138 (2002).

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Title: Talenti-type comparisons and isoperimetric inequalities in Riemannian manifolds.

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Abstract:

The establishment of Talenti-type results for the symmetrization of the solution to a Poisson equation on a smooth domain D within a Riemannian manifold (M, g) is linked to the existence of an isoperimetric inequality satisfied by the domains of the manifold. This relationship has been previously explored in [Mc], [CLD],[DH] and [MV], both in the non-compact and in the compact setting.

To date, knowledge regarding comparisons in spaces of negative curvature is limited, (see McDonald's work [Mc] on hyperbolic spaces). The primary difficulty lies in the fact that no isoperimetric inequality has been established in a general way for spaces of negative curvature.

In this work, we present an outline of the proof for a series of Talenti-type comparisons for complete and non-compact Riemannian manifolds that satisfy one of the following conditions:

- They possess a positive isoperimetric constant.
- Their isoperimetric profile is under control in some sense.

In the non-compact case, these hypotheses encompass the settings studied in [Mc], [CLD], and [DH], and include moreover Cartan-Hadamard manifolds.

References

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