On Flag Codes in Network Coding

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Abstract

Network coding is a part of information theory that describes a method to maximize the rate of a network, which is modeled by a directed acyclic multigraph, with one or multiple sources and multiple receivers. In this setting, the transmitted messages (*codewords*) are vector subspaces of a given vector space $\mathbb{F}_q^n$, where $\mathbb{F}_q$ is the finite field of q elements and a *subspace code* is just a collection $\mathcal{C}$ of vector subspaces of $\mathbb{F}_q^n$. When all subspaces of $\mathcal{C}$ have the same dimension, we say that $\mathcal{C}$ is a *constant dimension code*. The minimum distance $d(\mathcal{C})$ of $\mathcal{C}$ is computed in the usual way by using a metric called *subspace distance* in the set of all subspaces of $\mathbb{F}_q^n$.

In the context of *multi-shot network coding*, that is, when the subspace channel is used several times, *flag codes* were recently introduced as an extension of constant dimension codes. In this setting, codewords are sequences of nested subspaces (flags) of $\mathbb{F}_q^n$.

In this talk we will explain the basic characteristics of this type of codes, starting from subspace codes. We will focus, in particular, on characterising flag codes of maximum distance and we will give some examples of specific constructions by using the well-known transitive action of Singer groups on the sets of lines and hyperplanes.

Keywords: Network coding, subspace codes, flag codes, orbit codes, Singer groups