Stochastic processes to model growth phenomena

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Sigmoidal growth curves, such as the Gompertz, logistic, and Korf models, describe the characteristic S-shaped pattern observed in various biological and ecological processes (cf. Di Crescenzo and Spina [1], Román-Román *et al.* [2], Di Crescenzo and Paraggio [3]). These models depict initial slow growth, followed by rapid expansion, and eventually reaching a plateau as the system approaches its carrying capacity. Although each model offers unique insights into population dynamics, making them valuable tools in fields like epidemiology, ecology, and agriculture, such growth curves overlook random fluctuations typical of real-world dynamics. To address this limitation many efforts have been realized to introduce random models related to these curves. Among them, stochastic processes, such as birth-death processes or diffusion processes, stand out. Most of the times, such stochastic processes are constructed in such a way that their expected values coincide with the corresponding growth curves (cf. Román-Román *et al.* [4], Di Crescenzo *et al.* [5] and [6]). This contribution presents a survey of prominent sigmoidal curves and their stochastic counterparts, alongside real-world applications and simulation studies aimed at bridging theory with practice. The aforementioned topics constitute a research area in which the late lamented colleague Professor Román-Román made a notable and significant contribution.

References

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