REGULARITY ESTIMATES FOR A CLASS OF NONLOCAL EQUATIONS ARISING FROM DISCRETE STOCHASTIC PROCESSES

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In the recent years, the connection between nonlocal equations and discrete stochastic processes has received an increasing attention. This interplay can be described by a *dynamic programming equation*, which is stated as the nonlocal equation

$$\int_{B_1} \frac{u(x+\varepsilon y)-u(x)}{\varepsilon^2}\, d\mu_x(y) = f(x), \qquad x \in \Omega,$$

where $\varepsilon > 0$ and $x \mapsto \mu_x$ is any (non necessarily continuous) choice of symmetric probability measures on B_1 satisfying certain uniform ellipticity condition.

The importance, among others, of this equation lies in the fact that its solutions approximate a viscosity solution of a PDE as $\varepsilon \to 0$. This allows to explore new regularity techniques for solutions of PDEs by obtaining regularity estimates that hold uniformly for every sufficiently small $\varepsilon > 0$.

We present a proof of an asymptotic Hölder estimate and Harnack inequality for solutions of dynamic programming equations with bounded and measurable increments, an analogous result to the celebrated Krylov-Safonov regularity method for non-divergence form elliptic equations. The results also generalize to functions satisfying Pucci-type inequalities for discrete extremal operators, which allow to extend the result to a wider class of equations.

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