# Rigidity results for the capillary overdetermined problem

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**Abstract**. In this paper we obtain rigidity results for bounded positive solutions of the general capillary overdetermined problem

$$\begin{cases} \operatorname{div}\left(\frac{\nabla u}{\sqrt{1+|\nabla u|^2}}\right) + f(u) = 0 & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \\ \partial_\nu u = \kappa & \text{on } \partial\Omega, \end{cases} \tag{1}$$

where $f$ is a given $C^1$ function in $\mathbb{R}$, $\nu$ is the exterior unit normal, $\kappa$ is a constant and $\Omega \subset \mathbb{R}^n$ is a $C^1$ domain. Our main theorem states that if $n = 2, \kappa \neq 0$, $\partial\Omega$ is unbounded and connected, $|\nabla u|$ is bounded and there exists a nonpositive primitive $F$ of $f$ such that $F(0) \geq \left(1+\kappa^2\right)^{-\frac{1}{2}} - 1$, then $\Omega$ must be a half-plane and $u$ is a parallel solution. In other words, under our assumptions, if a capillary graph has the property that its mean curvature depends only on the height, then it is the graph of a one dimensional function. We also prove the boundedness of the gradient of solutions of (1) when $f'(u) < 0$. Moreover we study a Modica type estimate for the overdetermined problem (1) that allows us to prove that, unless $\Omega$ is a half-space, the mean curvature of $\partial\Omega$ is strictly negative under the assumption that $\kappa \neq 0$ and there exists a nonpositive primitive $F$ of $f$ such that $F(0) \geq \left(1+\kappa^2\right)^{-\frac{1}{2}} - 1$. Our results have an interesting physical application to the classical capillary overdetermined problem, i.e., the case where $f$ is linear.

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