

# Coloquio IMAG

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**The Beurling  
theorem for finite  
index shifts and  
the invariant  
subspace problem**

Lugar: Salón de  
grados. Facultad  
de Ciencias

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21/3/25  
12:30 h

**Abstract:** In this talk, I will present a generalization of the “concrete” form of Beurling’s theorem for the shift on the direct finite sum of  $H^2$ . I will show that every closed invariant subspace is given, up to multiplication by an inner function, by the intersection of a finite number of what we call “determinantal spaces”—which, roughly speaking, are the preimages of shift-invariant subspaces of  $H^2$  by a linear operator intertwining the shifts and constructed through a determinantal operator. The concreteness of such a structure theorem allows us to prove directly, as in the classical Beurling theorem, that the only non-trivial maximal closed shift-invariant subspaces are of codimension one. Using the universality of the (backward) shift in the class of operators with defect less than or equal to the index of the shift, this allows to obtain prove that every bounded linear operator from a Hilbert space into itself whose defect is finite has a non-trivial closed invariant subspace. The talk is based on a joint work with Eva Gallardo-Gutierrez