Nonlinear elliptic equations with Hardy potential, lower order term and with L^1 Data

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Abstract

In this talk we will consider the nonlinear elliptic problems whose prototype is:

$$\begin{cases} -div(|\nabla u|^{p-2}\nabla u + c_0(x)|u|^{\gamma-1}u) = \lambda \frac{|u|^{s-1}u}{|x|^p} + f & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases} \tag{1}$$

where Ω is any bounded open subset of $\mathbb{R}^N$, $N > 2$, $1 < p < N$, $0 \leq \gamma \leq p-1$, $0 \leq s < (p-1)(1-\frac{p}{N})$, $c_0(x)$ belongs to $L^{\frac{N}{p-1}}(\Omega)$ and f belongs to $L^1(\Omega)$. I will present an existence result of renormalized solution for the problem whose prototype is (1) with nonlinear and no-coercive operator, L^1-data and this for all $\lambda \geq 0$, without adding a suitable term. The main difficulties which arise in proving existence results for solution to (1) are the lack of coercivity due to the term $-div(c_0(x)|u|^{\gamma-1}u)$, and the effect of the singular term (the Hardy potential) that creates, in general, an obstruction to the existence of a solution. Using the framework of renormalized solution, is motivated by the lack of regularity of the weak solution also the distributional formulation is not strong enough to provide uniqueness. This notion was introduced by DiPerna and Lions [3] for the study of Boltzmann equations (see also [1, 4]). A large number of papers was then devoted to the study the existence of renormalized solution of parabolic problems with rough data under various assumptions and in different contexts: for a review on classical results, see for instance [2].

keywords: Nonlinear elliptic equation, Hardy potential, Renormalized solutions, Lower order terms,L1-data.

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