SEMINARIO GOYA
10 DECEMBER

Seminario 1. IMAG. Universidad de Granada

Link: https://meet.google.com/exr-weec-xoy

First talk: Seminario 1. 18:00h
Author: Ana Foulquié.
Título: Random walks through multiple orthogonal polynomias
Summary: In this talk we will establish the connection between non-negative Jacobi matrices, multiple orthogonal polynomials and random walks. We will give a general strategy for constructing a pair of stochastic matrices, dual to each other. The corresponding Markov chain will allow, in one transition, to reach the N-th previous states, to remain in the state or reach for the immediate next state. The dual Markov chains allow, in one transition, to reach for the N-th next states, to remain in the state or reach for the immediately previous state.

We will extend the Karlin-MacGregor representation formula to both dual random walks, and applied to the corresponding generating functions and first passage distributions.

We will take the Jacobi-Piñeiro multiple polynomials as a case study, and we will present the explicit formula for the type I Jacobi-Piñeiro polynomials and we will show the region of the parameters where the corresponding Jacobi matrix is non-negative. Examples of Recurrent and transient Jacobi-Piñeiro random walks will be shown.

This is a joint work with Amílcar Branquinho, Coimbra University, Juan E. Fernández-Díaz, Aveiro University and Manuel Mañas, Complutense University of Madrid.

Second talk: Seminario 1. 19:00h
Author: Miguel A Piñar (with Cleonice F. Bracciali).
Título: On Symmetric Orthogonal Polynomials.
Summary: We study families of multivariate orthogonal polynomials with respect to the symmetric weight function in d variables

$$B_\gamma(\mathtt{x}) = \prod_{i=1}^{d} w(x_i) \prod_{i<j} |x_i - x_j|^{2\gamma+1}, \quad \mathtt{x} \in (a,b)^d,$$

for $\gamma > -1$, where $w(t)$ is an univariate weight function in $t \in (a,b)$ and $\mathtt{x} = (x_1, x_2, \ldots, x_d)$ with $x_i \in (a,b)$. Using the change of variables $\mathtt{x} = (x_1, x_2, \ldots, x_d) \mapsto \mathtt{u} = (u_1, u_2, \ldots, u_d)$ where, u_r are the **r-th elementary symmetric functions** we study multivariate orthogonal polynomials in the variable $\mathtt{u}$ associated with the weight function $W_\gamma(\mathtt{u})$ defined by means of $W_\gamma(\mathtt{u}) = B_\gamma(\mathtt{x})$. For the new weight function, the domain is described in terms of the discriminant of the polynomial

having x_i, $i = 1, 2, \ldots, d$, as its zeros and in terms of the associated Sturm sequence. Obviously, generalized classical orthogonal polynomials as defined by Lassalle [2, 3, 4] and Macdonald [5] are included in our study. Choosing the univariate weight function as the Hermite, Laguerre and Jacobi weight functions, we obtain the representation in terms of the variables u_r for the partial differential operators having the respective Hermite, Laguerre and Jacobi generalized multivariate orthogonal polynomials as the corresponding eigenfunctions. The case $d = 2$ coincides with the polynomials studied by Koornwinder in [1]. Finally, we present explicitly the partial differential operators for Hermite, Laguerre and Jacobi generalized polynomials in the cases $d = 2$ and $d = 3$.

References

[1] T. H. Koornwinder, *Orthogonal polynomials in two variables which are eigenfunctions of two algebraically independent partial differential operators I*, Indag. Math., **3**6 (1974), 48–58.

[2] M. Lassalle, *Polynômes de Jacobi généralisés*, C. R. Acad. Sci. Paris Sér. I Math. **312** (1991), 425–428.

[3] M. Lassalle, *Polynômes de Laguerre généralisés*, C. R. Acad. Sci. Paris Sér. I Math. **312** (1991), 725–728.

[4] M. Lassalle, *Polynômes de Hermite généralisés*, C. R. Acad. Sci. Paris Sér. I Math. **312** (1991), 579–582.

[5] I. G. Macdonald, *Hypergeometric Functions I*, arXiv:1309.4568 [math.CA]