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Simultaneous deconvolution of multiple auditory evoked potentials in a reduced representation space

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Slide 1 (Title and authors):

Good morning, thank you very much. I will present a procedure for multiple response deconvolution for auditory evoked potentials.

Slide 2 (Simple equations):

And I would like to describe the mathematics of the procedure. Here you can see a simple equation with one unknown (x). Easy to be solved. And here a system of two equations with two unknowns (x_1, x_2), that you probably remember from the high school.

Slide 3 (Matrix form of a system of equations):

Systems of equations can be represented as a matrix equation (this is particularly useful for large systems). Here we have the vector of unknowns $\mathbf{x}$, the matrix of coefficients A, and the vector of observations $\mathbf{y}$. And the solution requires the inversion of the matrix of coefficients A.

Slide 4 (Over-determined system of equations):

If we have more equations than unknowns, it is an over-determined system of equations, in general with no solution.

Slide 5 (Over-determined system with noise):

However, since the observations are always affected by noise, over-determined systems with many equations are very useful in order to cancel the effect of the noise.

Slide 6 (The LS solution):

In such system (much more equations than unknowns) the matrix of coefficients is not square, but rectangular, and cannot be inverted. However, the statistics provides the least squares solution (the solution that minimizes the error between the observation and the expected observation). The procedure is the following:

- A^T is the transpose of the matrix of coefficients.
- We apply the transpose A^T to the observation $\mathbf{y}$, that is $A^T\mathbf{y}$.
- We apply the transpose A^T to the matrix A, that is A^TA, which is a square matrix, and can be inverted.
- And the solution is the inverse of this square matrix $(A^TA)^{-1}$ applied to the product $A^T\mathbf{y}$.

Slide 7 (Convolution as a system of equations):

The recording of auditory evoked potentials is usually modeled as a convolutional process. The electroencephalogram $y(n)$ is the convolution of the stimulation sequence $s(n)$ and the response $x(n)$, and is affected by additive noise $n_0(n)$.

The deconvolution (i.e. the estimation of the response) is a system of equations where: the samples of the response are the unknowns $x(n)$; the stimulation sequence $s(n)$ provides the coefficients; and the electroencephalogram $y(n)$ are the observations. The problem can be described as a matrix equation, and the least squares solution is obtained as described before:

- The transpose of the matrix S^T is applied to the electroencephalogram $\mathbf{y}$, that is $S^T\mathbf{y}$.
- The transpose applied to the matrix provides the square matrix (S^TS) to be inverted.
- And we apply the inverse $(S^TS)^{-1}$ to $S^T\mathbf{y}$ to obtain the response $\hat{\mathbf{x}}$.

Note that the term $S^T\mathbf{y}$ represents the synchronous sum of the individual responses. If we divide $S^T\mathbf{y}$ by the number of stimuli in the sequence, and multiply $(S^TS)^{-1}$ by the same number, the solution can be equivalently obtained as a matrix operation, where $\mathbf{z}_0$ is the synchronous averaging of the individual responses, and the matrix to be inverted R_s is the autocorrelation matrix of the stimulation sequence.

So, the deconvolution procedure is: (a) we obtain the synchronous averaging $\mathbf{z}_0$; (b) we obtain the autocorrelation matrix R_s from the stimulation sequence; (c) we invert the autocorrelation matrix and apply it to the synchronous averaging.

And that's all, we have the least squares estimation of the response.

Slide 8 (Practical problems):

But we should pay attention to the invertibility of the autocorrelation matrix. If it is singular (null eigenvalues) or quasi-singular (close to zero eigenvalues) we will have problems (because the inverse matrix amplifies specific components of the noise... for this reason, periodic or quasi-resonant stimulation sequences should be avoided). Also, the matrix inversion requires a high computational load. Both problems are affected by the dimensionality of the problem (i.e., the number of unknowns, the length of the response).

Slide 9 (Dimensionality reduction):

This problems can be reduced if we can transform the vector of unknowns to a subspace with lower dimensionality. In that case, the transformation from the original representation to the reduced representation is described by an orthonormal projector V_r, and we can rewrite the matrix equation and apply the least squares solution.

And after some matrix algebra we obtain the solution (in the reduced representation space) as the product of the inverted autocorrelation matrix (in the reduced representation space) applied to the synchronous averaging (in the reduced representation space).

In other words, exactly the same, (but all the operations performed in the reduced representation space).

Slide 10 (Advantages of the dimensionality reduction):

And importantly, the matrix operations are performed with a reduced dimensionality. This provides a noise reduction in the solution, and also, this reduces the computational cost. Additionally, the dimensionality reduction alleviates the problems related to matrix invertibility and reduces the computational cost associated to estimation of eigenvalues (very useful for diagnosing the matrix invertibility).

Slide 11 (Trick for dimensionality reduction):

But, how can the dimensionality be reduced? In auditory evoked potentials it can be easily achieved with non-uniform sampling.

Let's see an example: if we are interested on brainstem, middle latency and cortical responses, with uniform sampling we need 1 second of response sampled at 10 kHz, and so the response requires 10.000 samples.

With non uniform sampling, we have around 5 oscillations per decade (waves I,II,III, IV-V, VII in ABR between 1 and 10 ms; three-four waves in MLR between 10 and 100 ms, three-four waves between 100 ms and 1 second). So we have to cover three decades (between 1 and 1000 ms), and sampling with 40 samples/decade is more than enough. And, all these auditory responses can be described with only 120 samples.

And since dimensionality is reduced from 10.000 to 120, all the problems related to matrix inversion are reduced.

Slide 12 (Multi response deconvolution):

Multiple response deconvolution can also be described as a matrix equation. In this example, we consider two responses. The electroencephalogram includes the contribution of two stimulation sequences each one convolved with its corresponding response.

We have a matrix equation including both responses, and the matrix equation can be compacted, by horizontal concatenation of the stimulation matrices and vertical concatenation of the responses.

Therefore, we have again a matrix equation with a "super stimulation matrix" and a "super response vector".

Slide 13 (Multi response solution):

The formal least squares solution is (always the same): the inverse of the super-autocorrelation matrix applied to the concatenation of the synchronous averaging vectors.

Slide 14 (Dimensionality):

The formal solution is not difficult. But the dimensionality is multiplied by the number of responses.

Slide 15 (Multi response deconvolution in reduced space):

Similarly, the dimensionality can be reduced, and the multi-response deconvolution can be performed in a reduced representation space.

Slide 16 (Summary):

In summary, here we can see the dimensionality associated to the matrix inversion in single- or multi-response deconvolutions performed in the compete or the reduced representation space.

Slide 17 (Summary - example):

So, in the example, conventional deconvolution has a dimensionality 10.000. In a reduced representation space, we can reduce the dimensionality down to 120. In a multi-response deconvolution with 10 responses, the dimensionality grows up to 100.000. Multi-response in a reduced representation space moves to 1.200 components.

A (10.000 x 10.000) matrix is difficult to be inverted with nowadays computers; a (120 x 120) one this is very easy; a (1.200 x 1.200) one this is relatively easy. However, a (100.000 x 100.000) matrix is impossible to be inverted with conventional computers, because of memory overflow errors.

Slide 18 (Experiments):

We have evaluated multi-response deconvolution with auditory evoked potentials using clicks, presented at a random stimulation level between 0 and 80 dB, where the multi-response categorization is based on the stimulation level.

The response duration was 200 ms. The number of dimensions is 3277 in the complete representation space and 91 in the reduced representation space.

Slide 19 (Details):

We have performed different categorizations based on the stimulation level.

Slide 20 (Results - 1):

In these figures we can see the results with 4 categories. We can see the multi-response solution in intervals of 20 dB, as a function of the latency. Latency is logarithmically scaled. We can see the auditory brainstem responses, and the middle latency responses. In the left side the deconvolution in the reduced representation space, and in the right side, in the complete representation space (more affected by noise). The deconvolution took 28 seconds in the original representation and 10 in the reduced representation space.

Slide 21 (Results - 2):

The difference is more important as the number of categories increases. With 8 categories (intervals of 10 dB) the execution time reduces from 5 minutes to 23 seconds.

Slide 22 (Results - 3):

With 16 categories (intervals of 5 dB) the deconvolution in the original representation is not possible due to memory overflow, and in the reduced representation it took less than 1 minute.

Slide 23 (Results - 4):

And we can increase the number of categories to 24 or 32 with a reasonable deconvolution time.

Slides 24, 25, 26 (Results - execution time):

[Not discussed in the presentation, due to available time]. These slides compare the computation time associated to the deconvolution and the estimation of eigen-values in the complete and the reduced representation space.

Slides 27, 28, 29, 30 (Results - 4 subjects):

In these slides we can see individual responses for several subjects: subject 1, subject 2, subject 3, subject 4.

Slide 31 (Effect of increasing the number of categories):

As we increase the number of categories, the responses are more affected by noise, the resolution in intensity improves, and the computational load increases, but multi-response deconvolution is possible in the reduced representation space.

We are working in a procedure for reducing the noise, inspired by a work presented by Philipp Spitzer (from Innsbruck) in the previous IERASG conference.

Slide 32 (High resolution in intensity):

And here we can see some preliminary results. From these noisy high resolution responses, it is possible to obtain these clean responses.

Slide 33 (High resolution in intensity, color figure):

The multi-level responses can be represented with this color figure. The vertical axis is the stimulation level, the horizontal axis is the latency, and amplitude of the waves is represented with colors, red for positive peaks, blue for negative peaks. The figure in the right side represents the responses in a reduced range of amplitudes (with better resolution in amplitude of the waves).

Slide 34 (Conclusions):

To summarize my presentation: (1) Multi-response deconvolution of auditory evoked potentials is possible in the reduced representation space: the fundamentals are not difficult; the practical problems are identified and solved. (2) In this work we have applied multi-response deconvolution with a simple categorization based on stimulation level. (3) However, multi-response deconvolution can be applied to a broad range of new experimental designs. (4) We think that these tools could be very useful for new clinical and research applications.

Slide 35 (The team and the web):

This is my research team, the projects supporting this research. All these results, with examples, code etc. are or will be available in our web, https://wpd.ugr.es/~sig.proc.audiology/ .

Slide 36 (Jose Carlos Segura Luna, in memoriam):

Finally, I would like to remember our college, professor Jose Carlos Segura Luna, who passed away last week. I want to thank him for his contributions in our team for so many years.

Thank you very much for your attention.