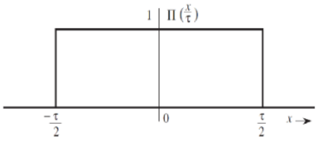
Procesamiento de Señales Biomédicas

Material de soporte para la resolución de Problemas del Tema 1

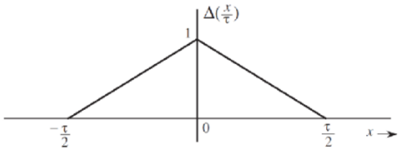
- **Función pulso rectangular.**

$$\prod(x/\tau) = \begin{cases} 1 & |x| \le \tau/2 \\ 0 & |x| > \tau/2 \end{cases}$$



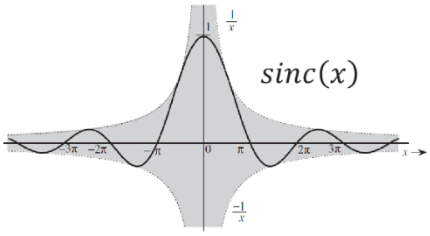
- **Función pulso triangular.**

$$\bigwedge(x/\tau) = \begin{cases} 1 - 2|x|/\tau & |x| \le \tau/2 \\ 0 & |x| > \tau/2 \end{cases}$$



- **Función $sinc(x)$.**

$$sinc(x) = \frac{sin(x)}{x}$$



	g(t)	G(f)	Condition
1	$e^{-at}u(t)$	$\dfrac{1}{a+j2\pi f}$	$a>0$
2	$e^{at}u(-t)$	$\dfrac{1}{a-j2\pi f}$	$a>0$
3	$e^{-a\|t\|}$	$\dfrac{2a}{a^2+(2\pi f)^2}$	$a>0$
4	$te^{-at}u(t)$	$\dfrac{1}{(a+j2\pi f)^2}$	$a>0$
5	$t^n e^{-at}u(t)$	$\dfrac{n!}{(a+j2\pi f)^{n+1}}$	$a>0$
6	$\delta(t)$	1	
7	1	$\delta(f)$	
8	$e^{j2\pi f_0 t}$	$\delta(f-f_0)$	
9	$\cos 2\pi f_0 t$	$0.5[\delta(f+f_0)+\delta(f-f_0)]$	
10	$\sin 2\pi f_0 t$	$j0.5[\delta(f+f_0)-\delta(f-f_0)]$	
11	$u(t)$	$\frac{1}{2}\delta(f)+\dfrac{1}{j2\pi f}$	
12	$\operatorname{sgn} t$	$\dfrac{2}{j2\pi f}$	

	g(t)	G(f)	Condition
13	$\cos 2\pi f_0 t\, u(t)$	$\frac{1}{4}[\delta(f-f_0)+\delta(f+f_0)]+\dfrac{j2\pi f}{(2\pi f_0)^2-(2\pi f)^2}$	
14	$\sin 2\pi f_0 t\, u(t)$	$\frac{1}{4j}[\delta(f-f_0)-\delta(f+f_0)]+\dfrac{2\pi f_0}{(2\pi f_0)^2-(2\pi f)^2}$	
15	$e^{-at}\sin 2\pi f_0 t\, u(t)$	$\dfrac{2\pi f_0}{(a+j2\pi f)^2+4\pi^2 f_0^2}$	$a>0$
16	$e^{-at}\cos 2\pi f_0 t\, u(t)$	$\dfrac{a+j2\pi f}{(a+j2\pi f)^2+4\pi^2 f_0^2}$	$a>0$
17	$\Pi\left(\frac{t}{\tau}\right)$	$\tau\,\operatorname{sinc}(\pi f\tau)$	
18	$2B\operatorname{sinc}(2\pi Bt)$	$\Pi\left(\dfrac{f}{2B}\right)$	
19	$\Delta\left(\frac{t}{\tau}\right)$	$\frac{\tau}{2}\operatorname{sinc}^2\left(\dfrac{\pi f\tau}{2}\right)$	
20	$B\operatorname{sinc}^2(\pi Bt)$	$\Delta\left(\dfrac{f}{2B}\right)$	
21	$\sum_{n=-\infty}^{\infty}\delta(t-nT)$	$f_0\sum_{n=-\infty}^{\infty}\delta(f-nf_0)$	$f_0=\dfrac{1}{T}$
22	$e^{-t^2/2\sigma^2}$	$\sigma\sqrt{2\pi}e^{-2(\sigma\pi f)^2}$	

Operation	g(t)	G(f)
Linearity	$\alpha_1 g_1(t)+\alpha_2 g_2(t)$	$\alpha_1 G_1(f)+\alpha_2 G_2(f)$
Duality	$G(t)$	$g(-f)$
Time scaling	$g(at)$	$\frac{1}{\|a\|}G\left(\frac{f}{a}\right)$
Time shifting	$g(t-t_0)$	$G(f)e^{-j2\pi f t_0}$
Frequency shifting	$g(t)e^{j2\pi f_0 t}$	$G(f-f_0)$
Time convolution	$g_1(t)*g_2(t)$	$G_1(f)G_2(f)$
Frequency convolution	$g_1(t)g_2(t)$	$G_1(f)*G_2(f)$
Time differentiation	$\dfrac{d^n g(t)}{dt^n}$	$(j2\pi f)^n G(f)$
Time integration	$\int_{-\infty}^{t} g(x)\,dx$	$\dfrac{G(f)}{j2\pi f}+\frac{1}{2}G(0)\delta(f)$

$$e^{\pm j\theta} = \cos\theta \pm j\sin\theta$$
$$e^{j2\alpha} + e^{j2\beta} = 2\cos(\alpha - \beta)e^{j(\alpha+\beta)}$$
$$e^{j2\alpha} - e^{j2\beta} = j2\sin(\alpha - \beta)e^{j(\alpha+\beta)}$$
$$\cos\theta = \frac{1}{2}(e^{j\theta} + e^{-j\theta}) = \sin(\theta + 90°)$$
$$\sin\theta = \frac{1}{2j}(e^{j\theta} - e^{-j\theta}) = \cos(\theta - 90°)$$
$$\sin^2\theta + \cos^2\theta = 1$$
$$\cos^2\theta - \sin^2\theta = \cos 2\theta$$
$$\cos^2\theta = \tfrac{1}{2}(1 + \cos 2\theta)$$
$$\cos^3\theta = \tfrac{1}{4}(3\cos\theta + \cos 3\theta)$$
$$\sin^2\theta = \tfrac{1}{2}(1 - \cos 2\theta)$$
$$\sin^3\theta = \tfrac{1}{4}(3\sin\theta - \sin 3\theta)$$
$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$
$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$$
$$\sin\alpha\sin\beta = \tfrac{1}{2}\cos(\alpha - \beta) - \tfrac{1}{2}\cos(\alpha + \beta)$$
$$\cos\alpha\cos\beta = \tfrac{1}{2}\cos(\alpha - \beta) + \tfrac{1}{2}\cos(\alpha + \beta)$$
$$\sin\alpha\cos\beta = \tfrac{1}{2}\sin(\alpha - \beta) + \tfrac{1}{2}\sin(\alpha + \beta)$$
$$A\cos(\theta + \alpha) + B\cos(\theta + \beta) = C\cos\theta - S\sin\theta = R\cos(\theta + \phi)$$
where
$$C = A\cos\alpha + B\cos\beta$$
$$S = A\sin\alpha + B\sin\beta$$
$$R = \sqrt{C^2 + S^2} = \sqrt{A^2 + B^2 + 2AB\cos(\alpha - \beta)}$$
$$\phi = \arctan\frac{S}{C} = \arctan\frac{A\sin\alpha + B\sin\beta}{A\cos\alpha + B\cos\beta}$$

- $A = |A| \cdot e^{j\varphi_A}$, $B = |B| \cdot e^{j\varphi_B}$
- $|A+B|^2 = |A|^2 + |B|^2 + 2 \cdot |A| \cdot |B| \cdot \cos(\varphi_A - \varphi_B)$
- $|A-B|^2 = |A|^2 + |B|^2 - 2 \cdot |A| \cdot |B| \cdot \cos(\varphi_A - \varphi_B)$

Inmediatas	*Cuasi inmediatas (con funciones)*
$\int x^n dx = \frac{x^{n+1}}{n+1} + k$ se suma 1 al expo y se divide por lo mismo	$\int f^n(x) \cdot f'(x)dx = \frac{f^{n+1}(x)}{n+1} + k$
$\int \frac{1}{x}dx = \ln\lvert x\rvert + k$	$\int \frac{f'(x)}{f(x)}dx = \ln\lvert f(x)\rvert + k$
$\int e^x dx = e^x + k$ se queda igual	$\int e^{f(x)} f'(x)dx = e^{f(x)} + k$
$\int a^x dx = \frac{a^x}{\ln a} + k$	$\int a^{f(x)} f'(x)dx = \frac{a^{f(x)}}{\ln a} + k$
$\int \text{sen}x dx = -\cos x + k$	$\int \text{sen}(f(x))f'(x)dx = -\cos(f(x)) + k$
$\int \cos x dx = \text{sen}x + k$	$\int \cos(f(x))f'(x)dx = \text{sen}(f(x)) + k$
$\int \frac{1}{\cos^2 x}dx = \int (1 + tg^2 x)dx = tgx + k$	$\int \frac{f'(x)}{\cos^2(f(x))}dx = \int (1 + tg^2(f(x))f'(x)dx = tg(f(x)) + k$
$\int \frac{1}{\text{sen}^2 x}dx = \int (1 + cotg^2 x)dx = -ctgx + k$	$\int \frac{f'(x)}{\text{sen}^2(f(x))}dx =$ $\int (1 + ctg^2(f(x))f'(x)dx = -ctg(f(x)) + k$
$\int \frac{1}{\sqrt{1-x^2}}dx = \text{arcsen}x + k$	$\int \frac{f'(x)}{\sqrt{1-f^2(x)}}dx = \text{arcsen}(f(x)) + k$
$\int \frac{1}{1+x^2}dx = \text{arctg}x + k$	$\int \frac{f'(x)}{1+f^2(x)}dx = \text{arctg}(f(x)) + k$