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On R_0 for populations with continuous structure

In a model of population growth, the basic reproduction number R_0 is defined as the expected number of children that an individual has throughout his life in a fixed environment (in an epidemiological model, as the expected number of new infections a newly infected individual will produce). In structured populations, when the birth event can happen in different individual states (of size, phenotype, spatial position, etc.) one talks of typical individual but it is not always clear what typical individual means and so, which is the expected number of children of a typical individual. On the other hand, in deterministic modeling we could be unsatisfied with a definition that involves a probability concept as expectation. Both drawbacks are solved defining R_0 as the spectral radius of the so-called first- generation operator, which maps a (distribution of) population to the (distribution of) population of their children along the whole life span of the former ([3]). Nevertheless, also this definition has some drawback since in some models the definition of the birth operator and that of the first-generator operator depend on the choice of the sometimes arbitrary concept of birth event ([2], [1]). When the structuring variable is discrete, i.e. when the population dynamics can be described by a system of odes, the first-generation operator is a matrix and there is no problem with the definition above leaving apart that it may be non-uniquely determined. However, in continuously structured populations with concentrated state at birth, the birth rate shows as a boundary condition instead of a bounded (birth) operator. This makes difficult to adapt to Banach spaces the above definition. We consider some examples where R_0 is defined as a limit of basic reproduction numbers of approximate models with distributed states at births for which the next generation operator can be defined as a bounded linear operator.

References

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