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Deconvolution for flexible recording of transient evoked potentials

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International Evoked Response Audiometry Study Group (IERASG)

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Signal Processing in Audiology

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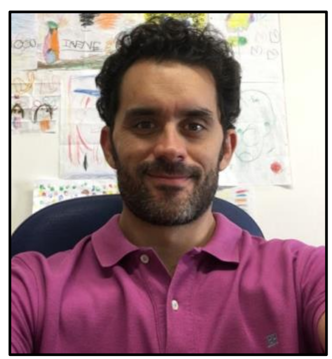

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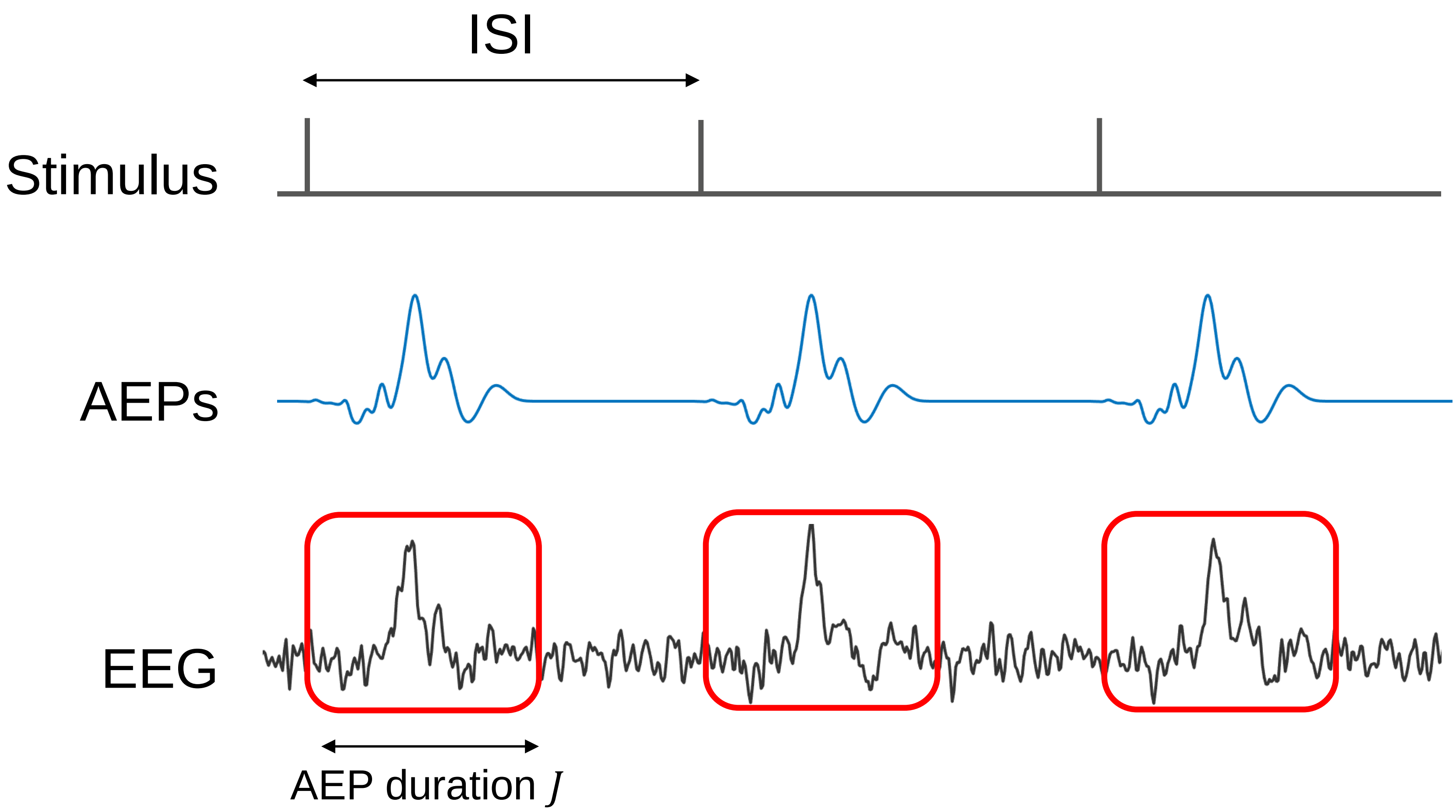
Conventional recording of AEPs



Conventional method → ISI > J

Matrix Deconvolution

The EEG as a *convolution* model

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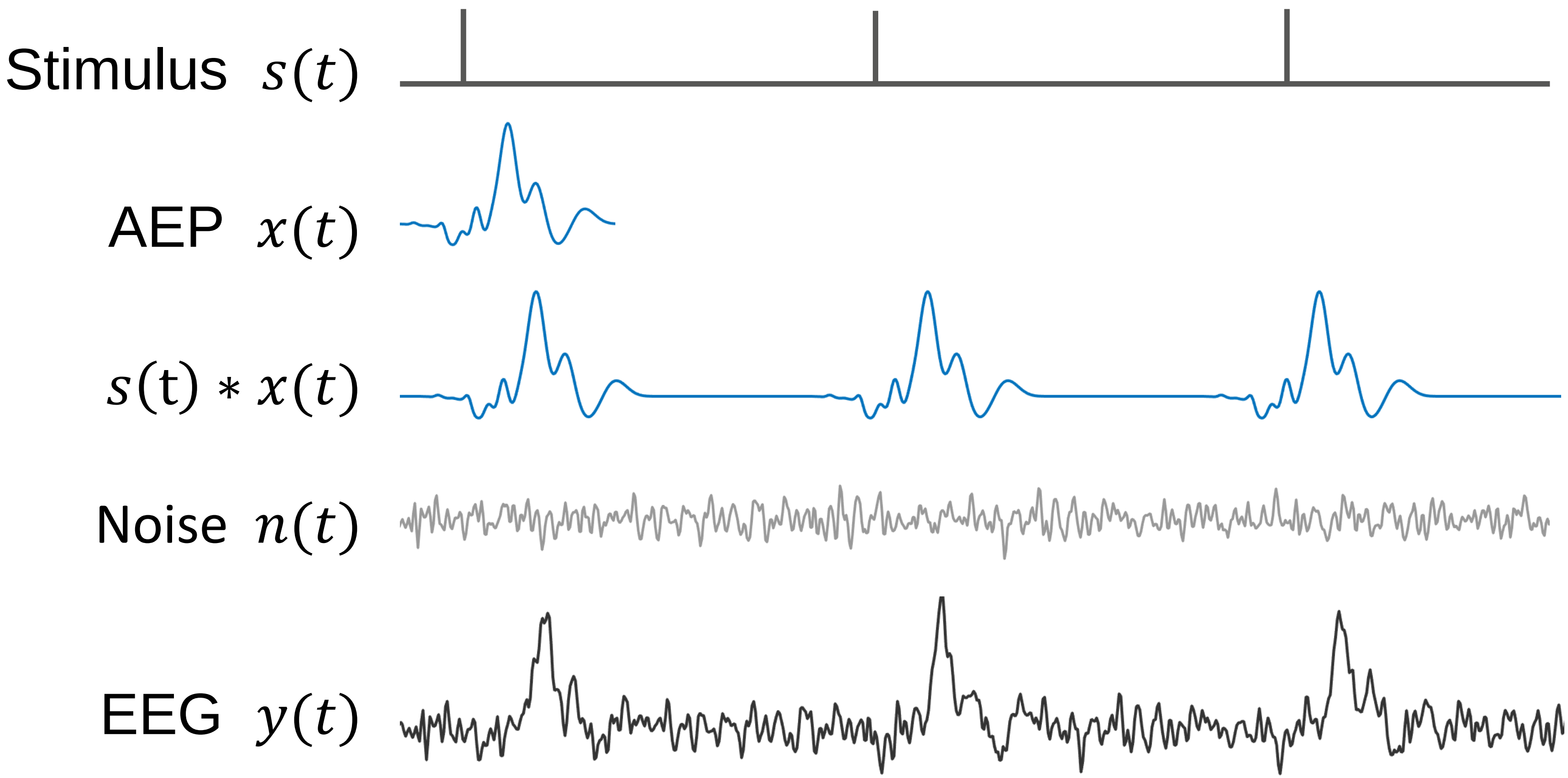
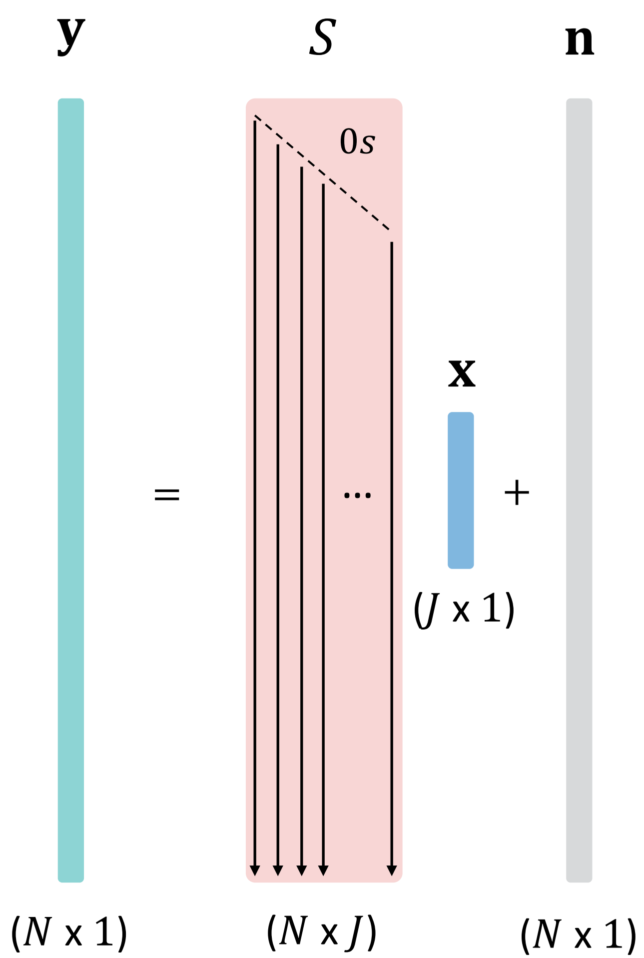
$$y(t) = s(t) * x(t) + n(t)$$



Matrix formulation

$$y(t) = s(t) * x(t) + n(t)$$

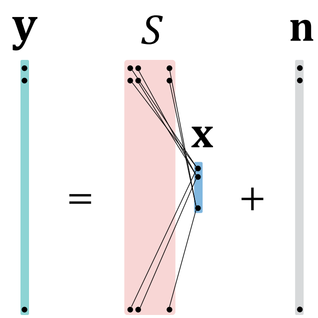
$$\mathbf{y} = S\mathbf{x} + \mathbf{n}$$



N – length of EEG
J – length of AEP
$J << N$

Matrix Deconvolution

$$\mathbf{y} = S\mathbf{x} + \mathbf{n}$$

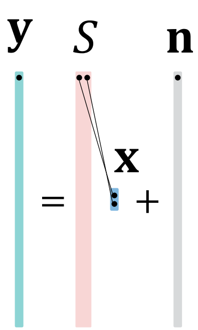


$$y_1 = S_{11}x_1 + S_{12}x_2 + \cdots + S_{1J}x_J + n_1$$
$$y_2 = S_{21}x_1 + S_{22}x_2 + \cdots + S_{2J}x_J + n_2$$
$$\vdots$$
$$y_N = S_{N1}x_1 + S_{N2}x_2 + \cdots + S_{NJ}x_J + n_N$$

N – length of EEG
J – length of AEP
J << N

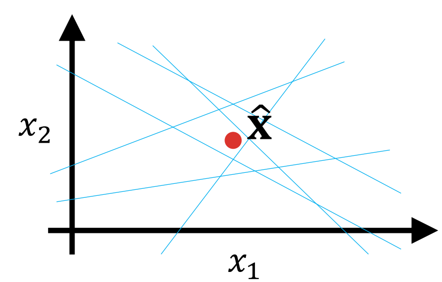
N equations, J unknowns

Let´s imagine an AEP of 2 samples (J = 2)



$$y_1 = S_{11}x_1 + S_{12}x_2 + n_1$$
$$y_2 = S_{21}x_1 + S_{22}x_2 + n_2$$
$$\vdots$$
$$y_N = S_{N1}x_1 + S_{N2}x_2 + n_N$$

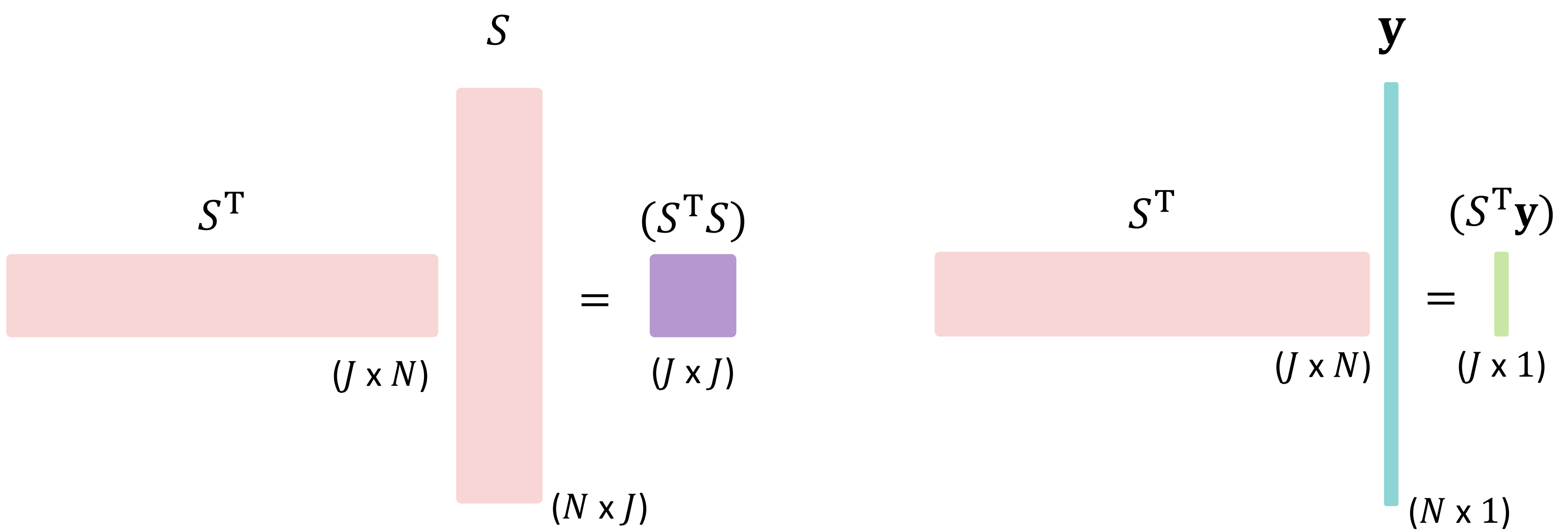
N equations, 2 unknowns



$$\hat{\mathbf{x}} = (S^{\mathrm{T}}S)^{-1}(S^{\mathrm{T}}\mathbf{y})$$

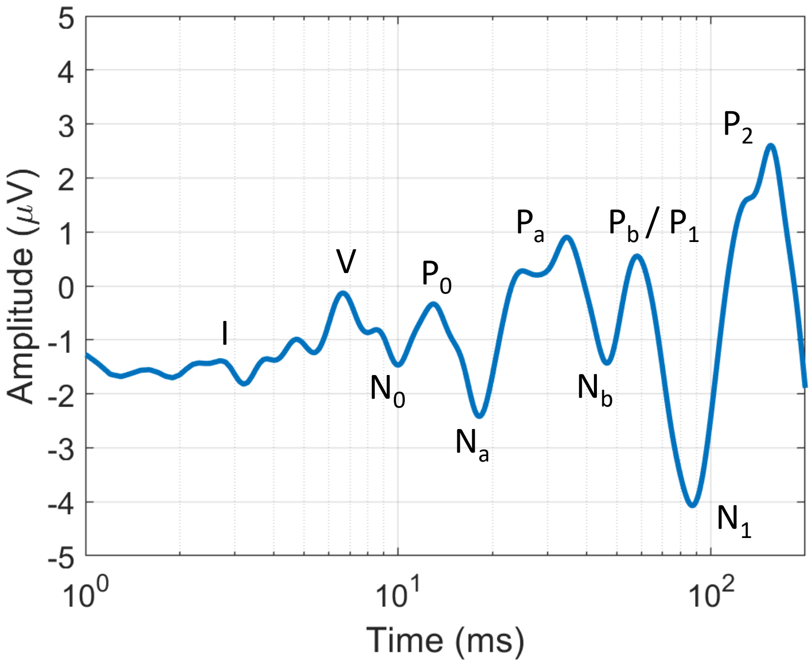
Matrix Deconvolution

$$\hat{\mathbf{x}} = (S^{\mathrm{T}}S)^{-1}(S^{\mathrm{T}}\mathbf{y})$$



N – length of EEG
J – length of AEP
$J << N$

Example of Matrix Deconvolution



$$\hat{\mathbf{x}} = (S^{\mathrm{T}}S)^{-1}(S^{\mathrm{T}}\mathbf{y})$$

- AEP of 200 ms @ 16,384 Hz
- $J = 3{,}277$ samples → $(S^{\mathrm{T}}S)_{(3277 \times 3277)}$
- How long does deconvolution take?
- 9 seconds

Multi-response Deconvolution

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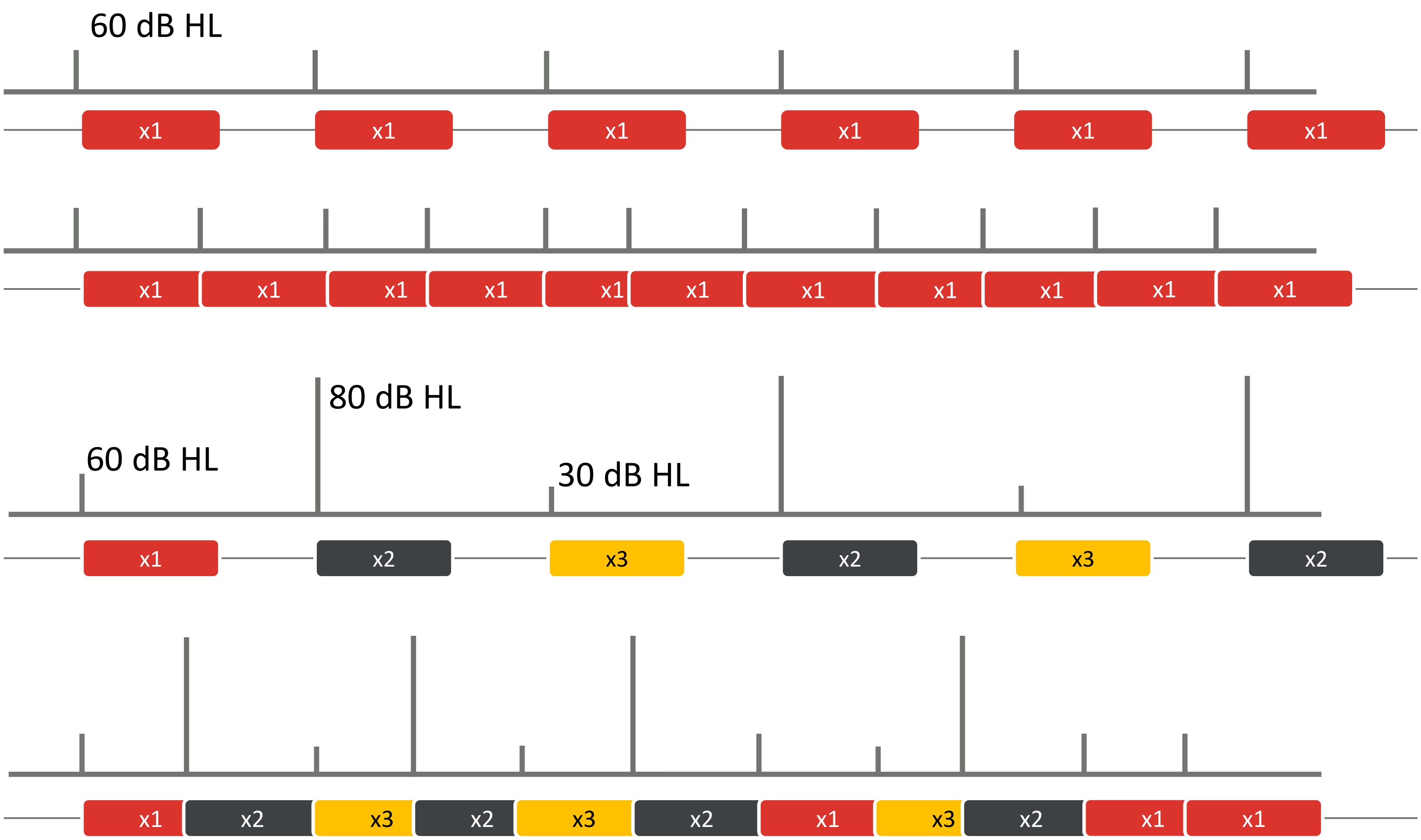

Multi-response Deconvolution

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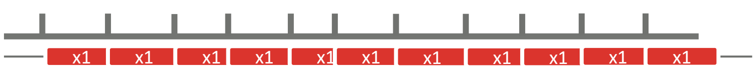
For 1 class

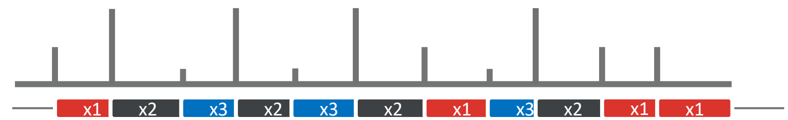


$$y(t) = s(t) * x(t) + n(t)$$

$$\hat{\mathbf{x}} = (S^{\mathrm{T}}S)^{-1}(S^{\mathrm{T}}\mathbf{y})$$

$$\underset{(J \times 1)}{\hat{\mathbf{x}}} = \underset{(J \times J)}{(S^{\mathrm{T}}S)^{-1}} \; \underset{(J \times 1)}{(S^{\mathrm{T}}\mathbf{y})}$$

For K different classes



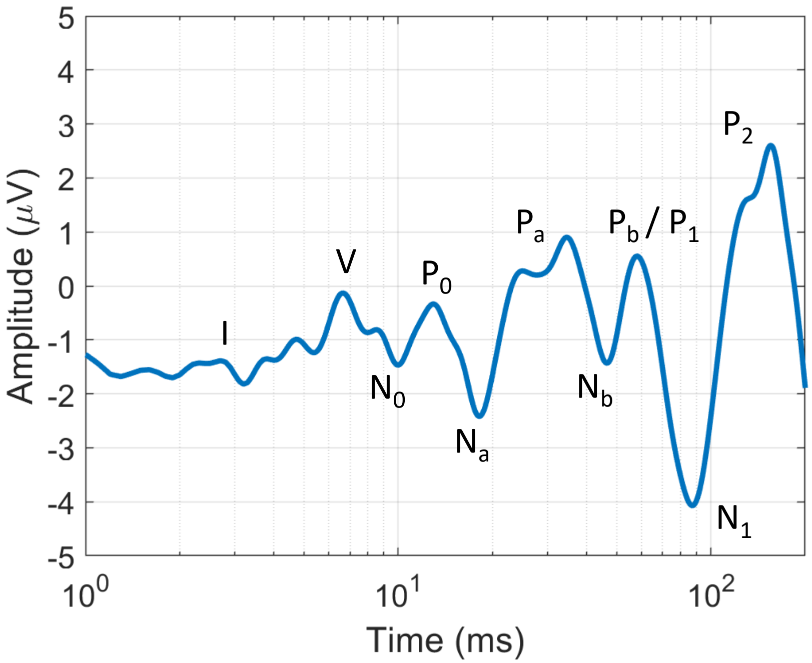
$$y(t) = s_1(t) * x_1(t) + s_2(t) * x_2(t) + \cdots + s_K(t) * x_K(t) + n(t)$$

$$\underset{(K \cdot J \times 1)}{\hat{\mathbf{x}}_{all}} = \underset{(K \cdot J \;\times\; K \cdot J)}{(S_{all}^T S_{all})^{-1}} \; \underset{(K \cdot J \times 1)}{(S_{all}^T \mathbf{y})}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_K \end{bmatrix} = \begin{bmatrix} S_1^T S_1 & S_1^T S_2 & \cdots & S_1^T S_K \\ S_2^T S_1 & S_2^T S_2 & \cdots & S_2^T S_K \\ \vdots & \vdots & \ddots & \vdots \\ S_K^T S_1 & S_K^T S_2 & \cdots & S_K^T S_K \end{bmatrix} \begin{bmatrix} S_1^T \mathbf{y} \\ S_2^T \mathbf{y} \\ \vdots \\ S_K^T \mathbf{y} \end{bmatrix}$$

(de la Torre et al., IERASG 2023)

Example of Multi-response Deconvolution



- AEPs of 200 ms @ 16,384 Hz → $J = 3{,}277$ samples
- $K = 10$ classes →$(S_{all}^T S_{all})_{(32,770\ x\ 32,770)}$
- $(S_{all}^T S_{all})_{(32,770\ x\ 32,770)}$ → 1,073,872,900 numbers * 8 bytes → 8,6 GB
- Deconvolution takes 1065 s
- For $K > 10$ classes, *Out-of-memory!*

Latency-Dependent Filtering & Downsampling

Representation of evoked potentials

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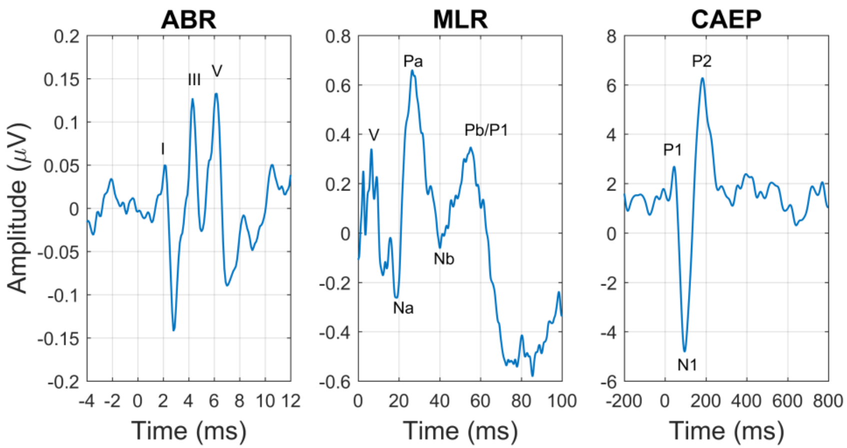
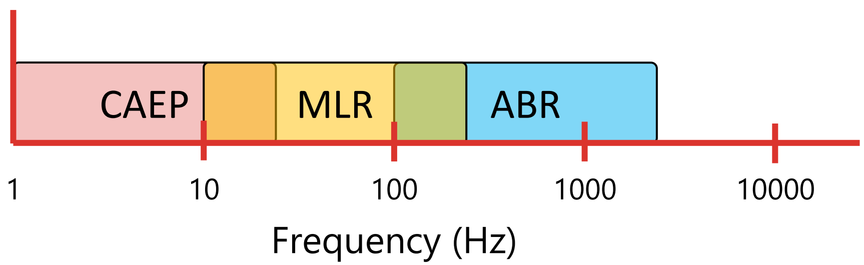
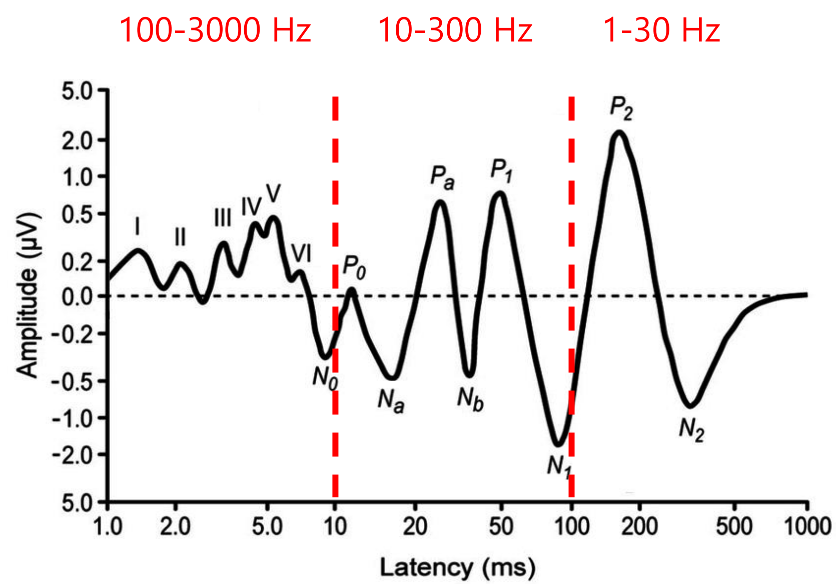
Conventional representation

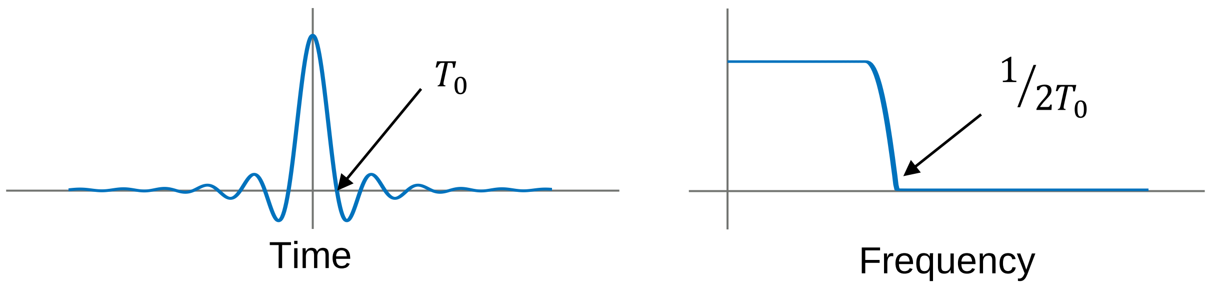


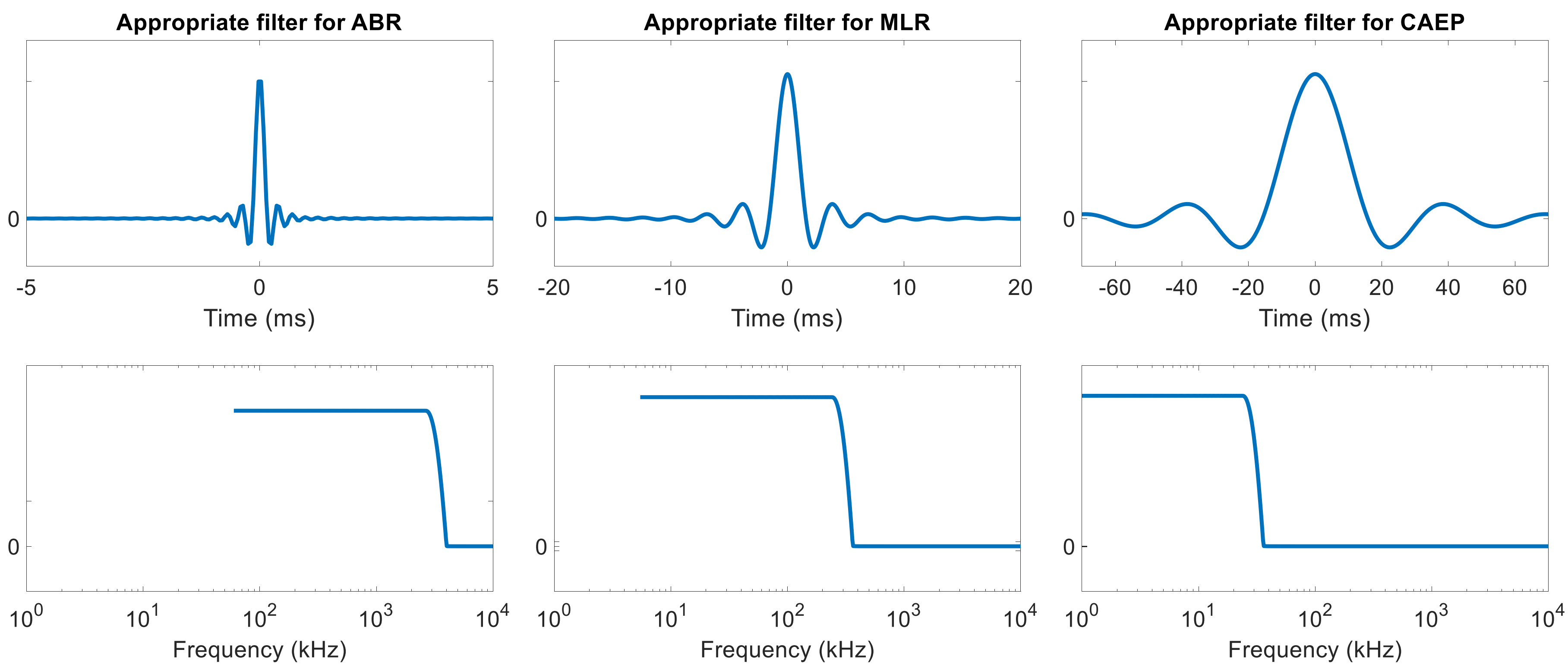


Desired representation



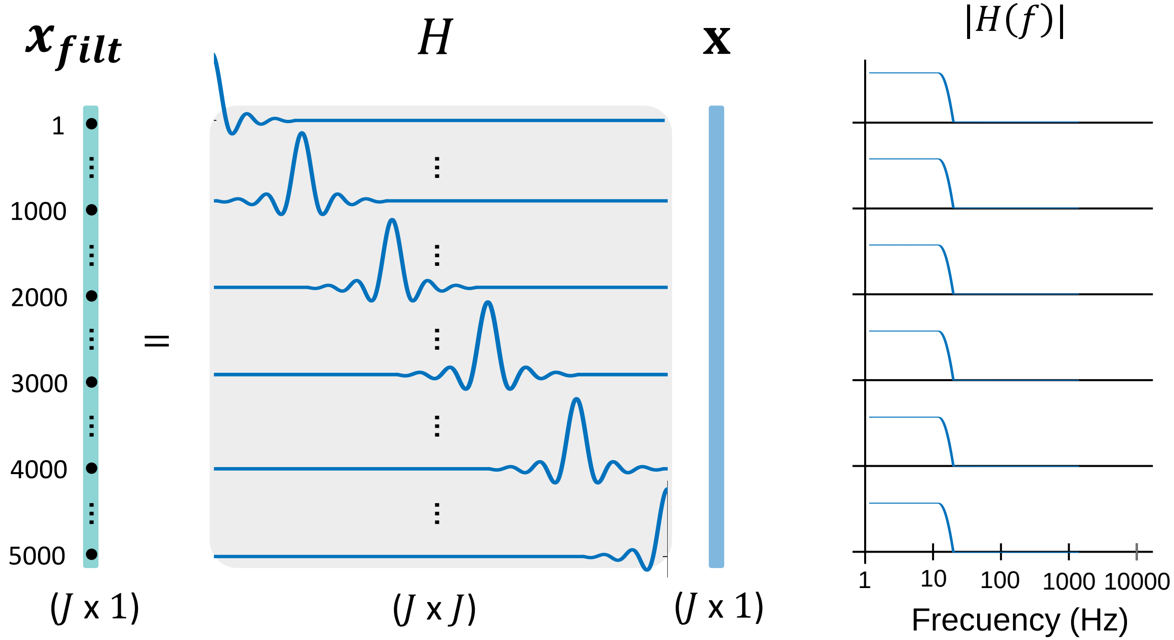
What is a filter like?





Conventional filtering

$$x_{filt}(n) = h(n) * x(n) \longrightarrow \mathbf{x}_{filt} = H\mathbf{x}$$



The same filter is used in all samples

Latency-dependent filtering

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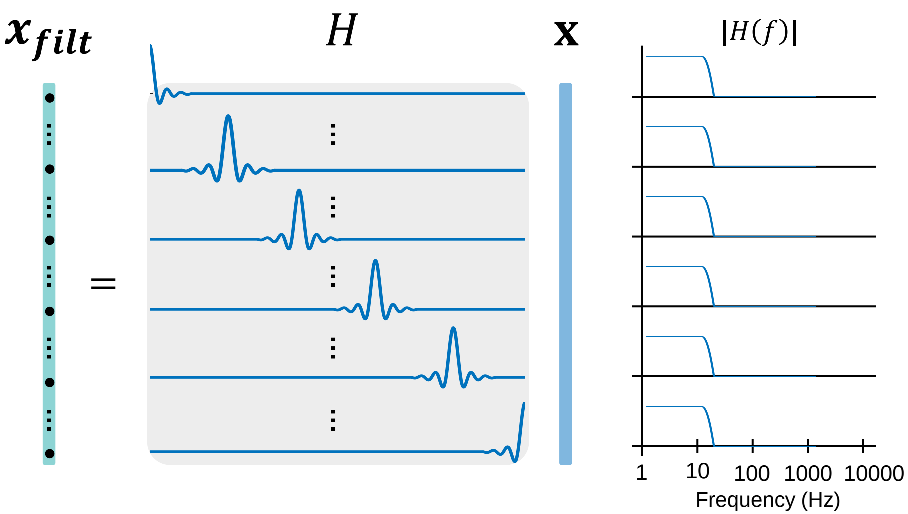
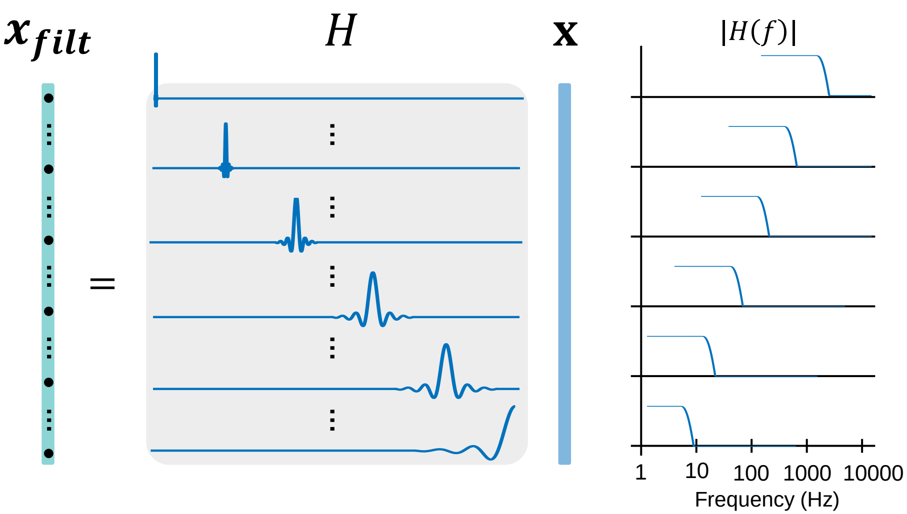
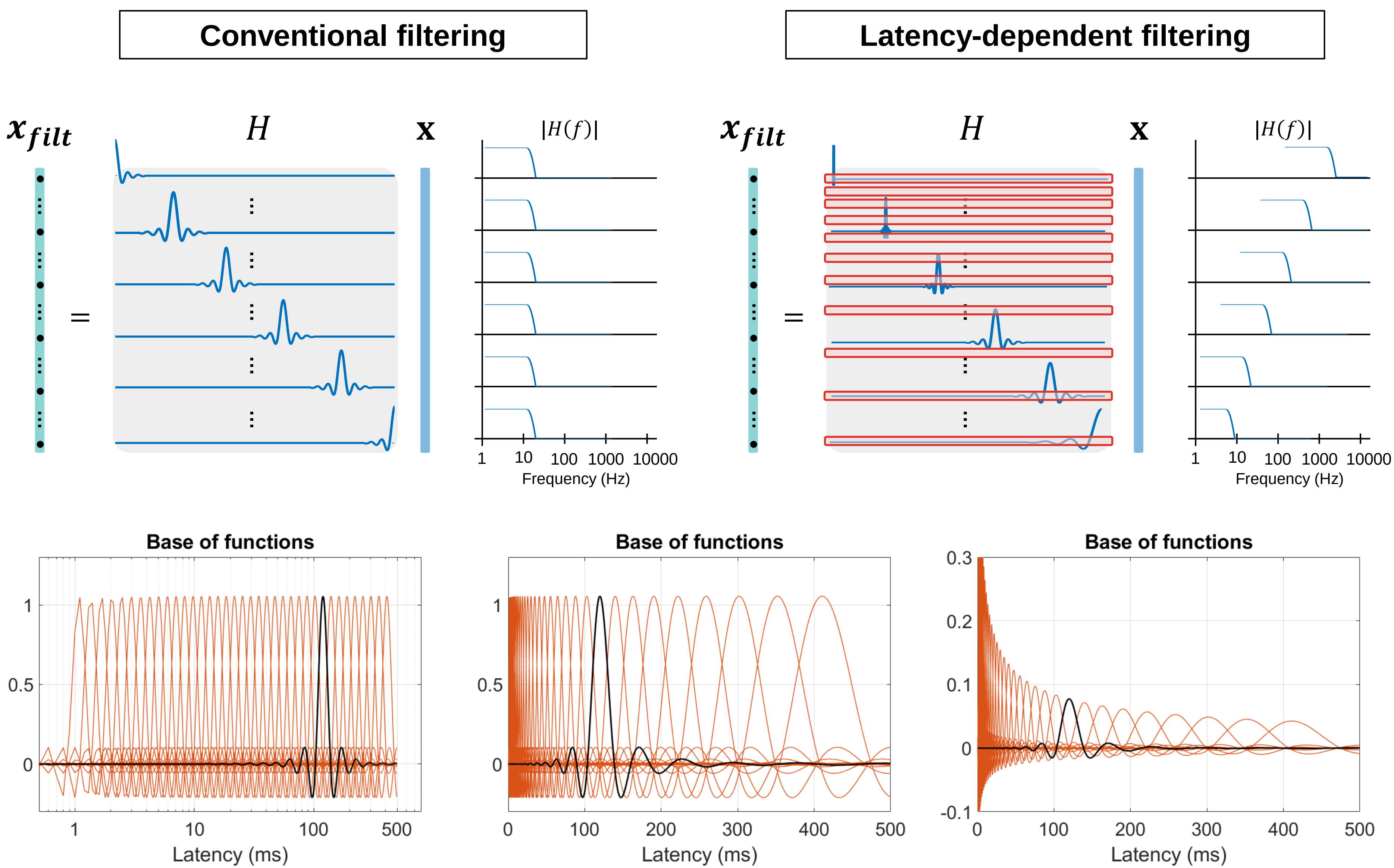
Conventional filtering



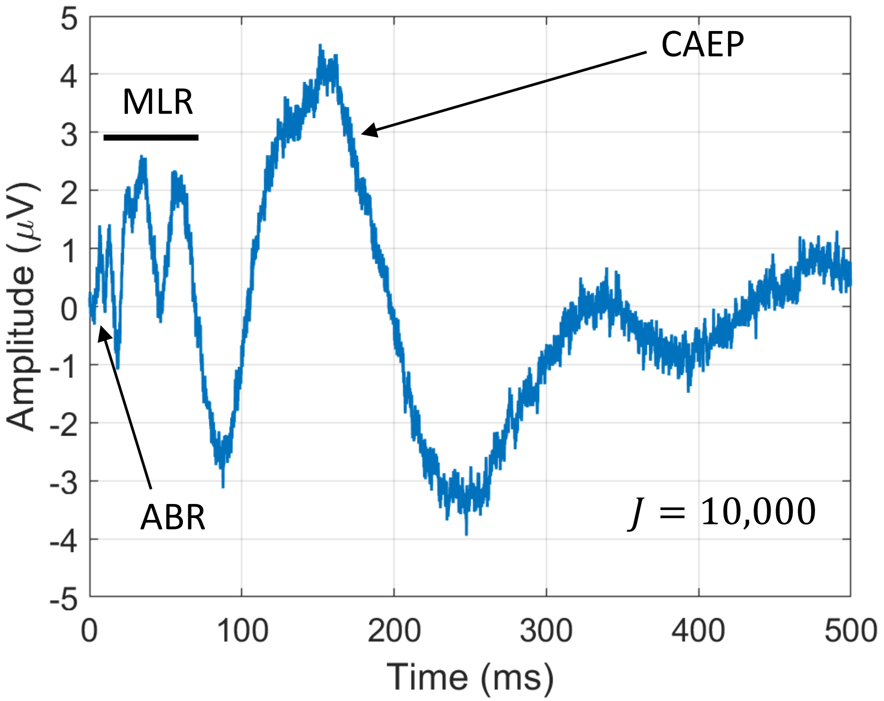
Latency-dependent filtering



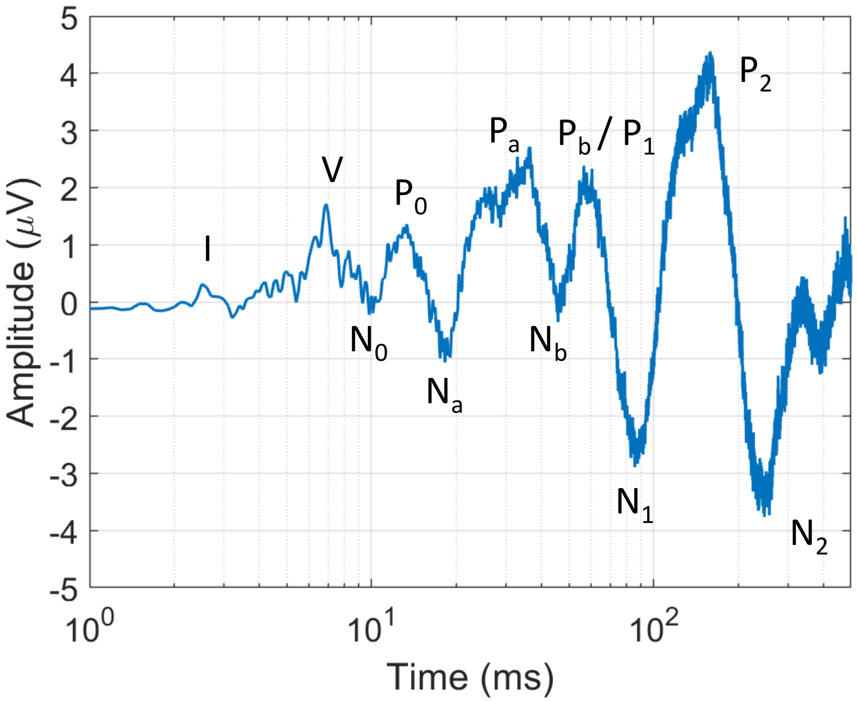
Latency-dependent filtering & DOWNSAMPLING



Latency-dependent filtering & downsampling (LDFDS)



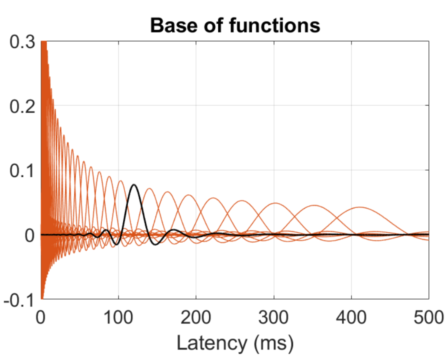
Latency-dependent filtering & downsampling (LDFDS)



- Project the AEP from the time domain onto the reduced space

$$\mathbf{x}_{red} = V \, \mathbf{x}$$

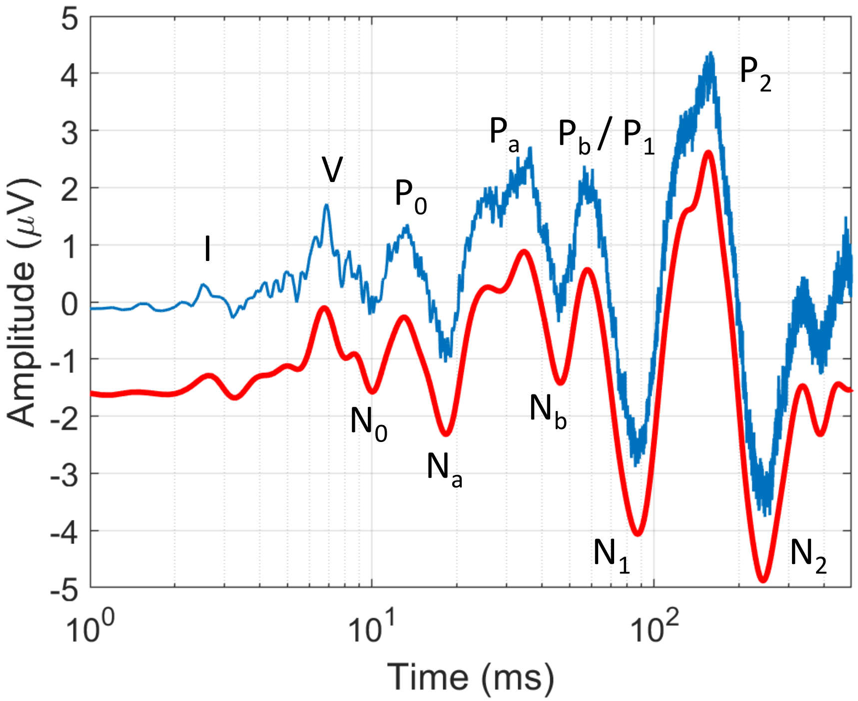
$\mathbf{x}_{red}$: (43 x 1) $\quad V$: (43 x 10,000) $\quad \mathbf{x}$: (10,000 x 1)



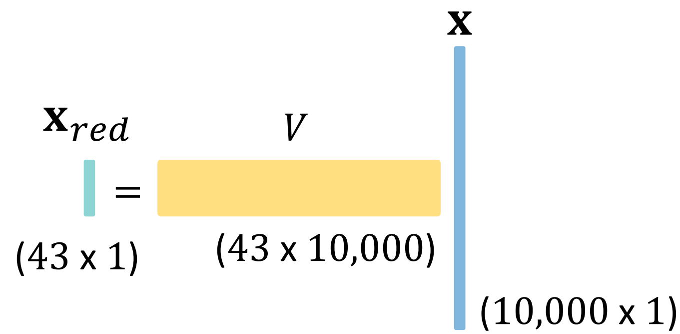
→ V

(43 x 10,000)

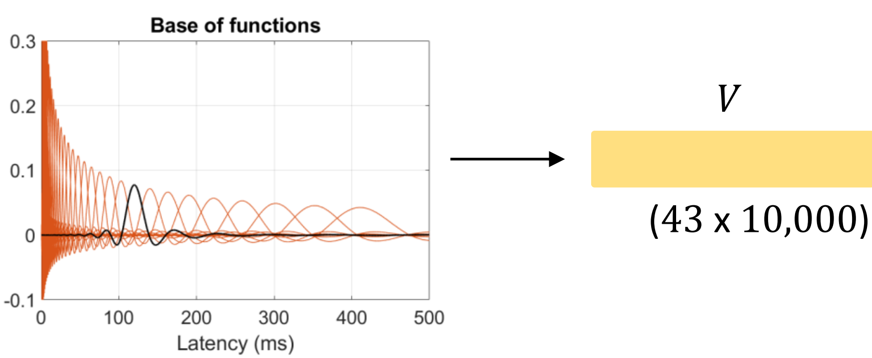
Latency-dependent filtering & downsampling (LDFDS)

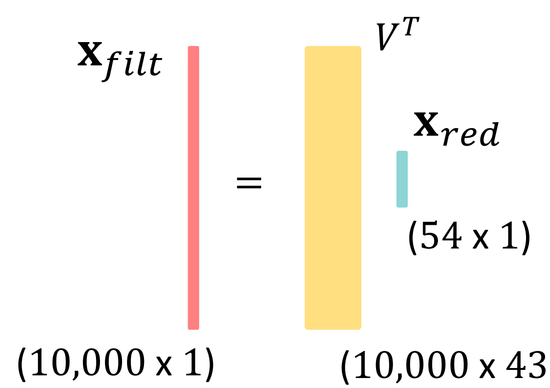


- Project the AEP from the time domain onto the reduced space



- Filter by projecting back the AEP from the reduced domain onto the time domain





Optimised Deconvolution

(performed in the subspace defined by LDFDS)

Optimised Matrix Deconvolution

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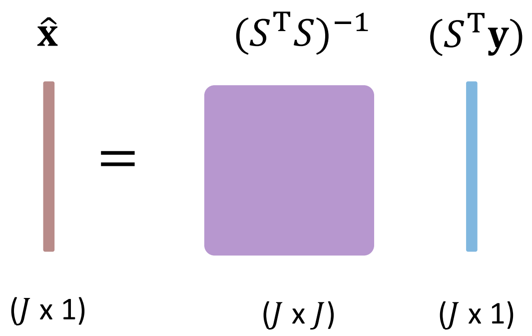
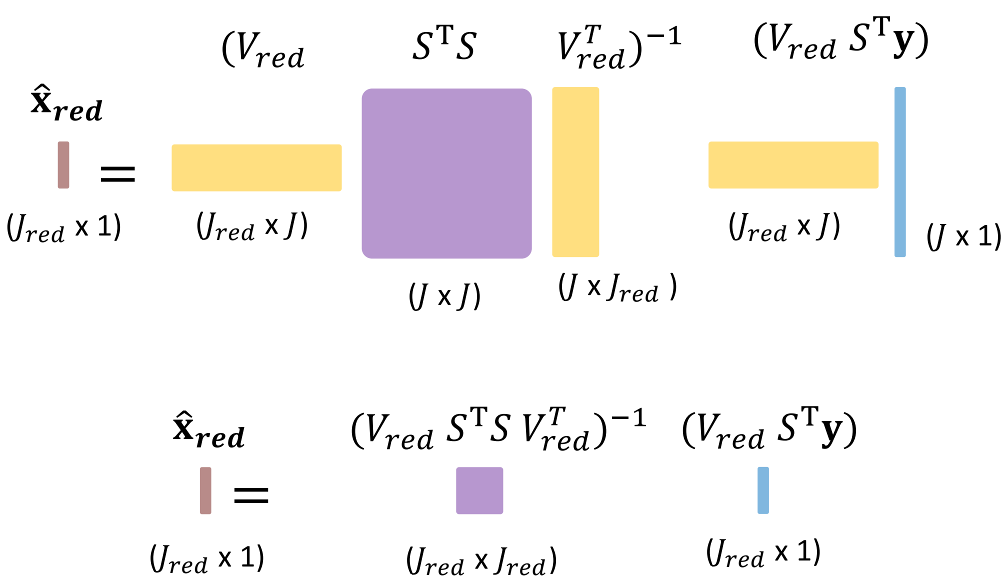
Matrix Deconvolution

$$\hat{\mathbf{x}} = (S^{\mathrm{T}}S)^{-1}(S^{\mathrm{T}}\mathbf{y})$$



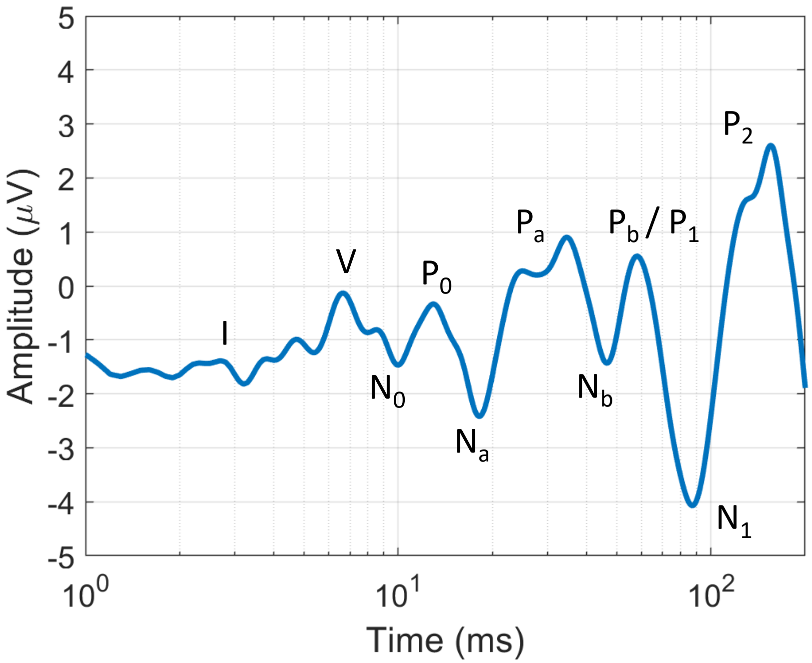
Subspace Constrained Matrix Deconvolution

$$\hat{\mathbf{x}}_{red} = (V_{red}\, S^{\mathrm{T}}S\, V_{red}^{T})^{-1}(V_{red}\, S^{\mathrm{T}}\mathbf{y})$$



$J_{red} \ll J$

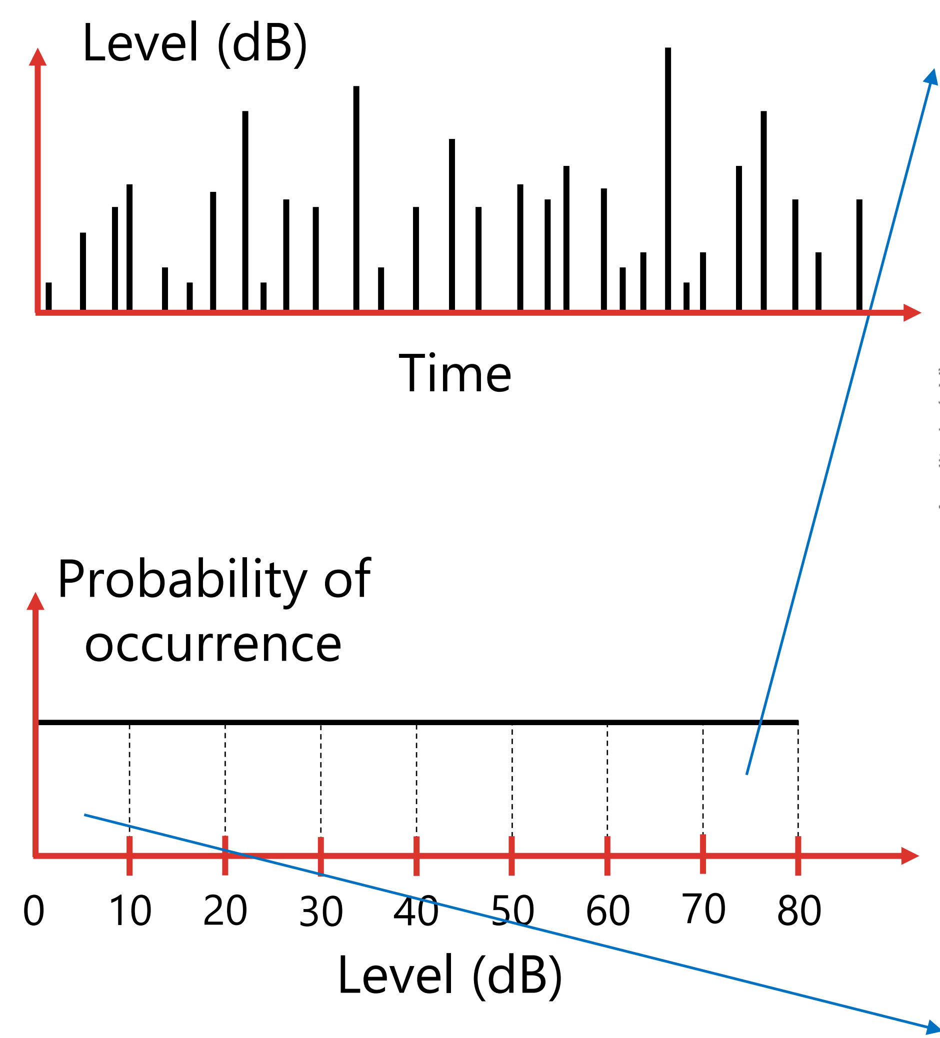
Example of Multi-response Deconvolution

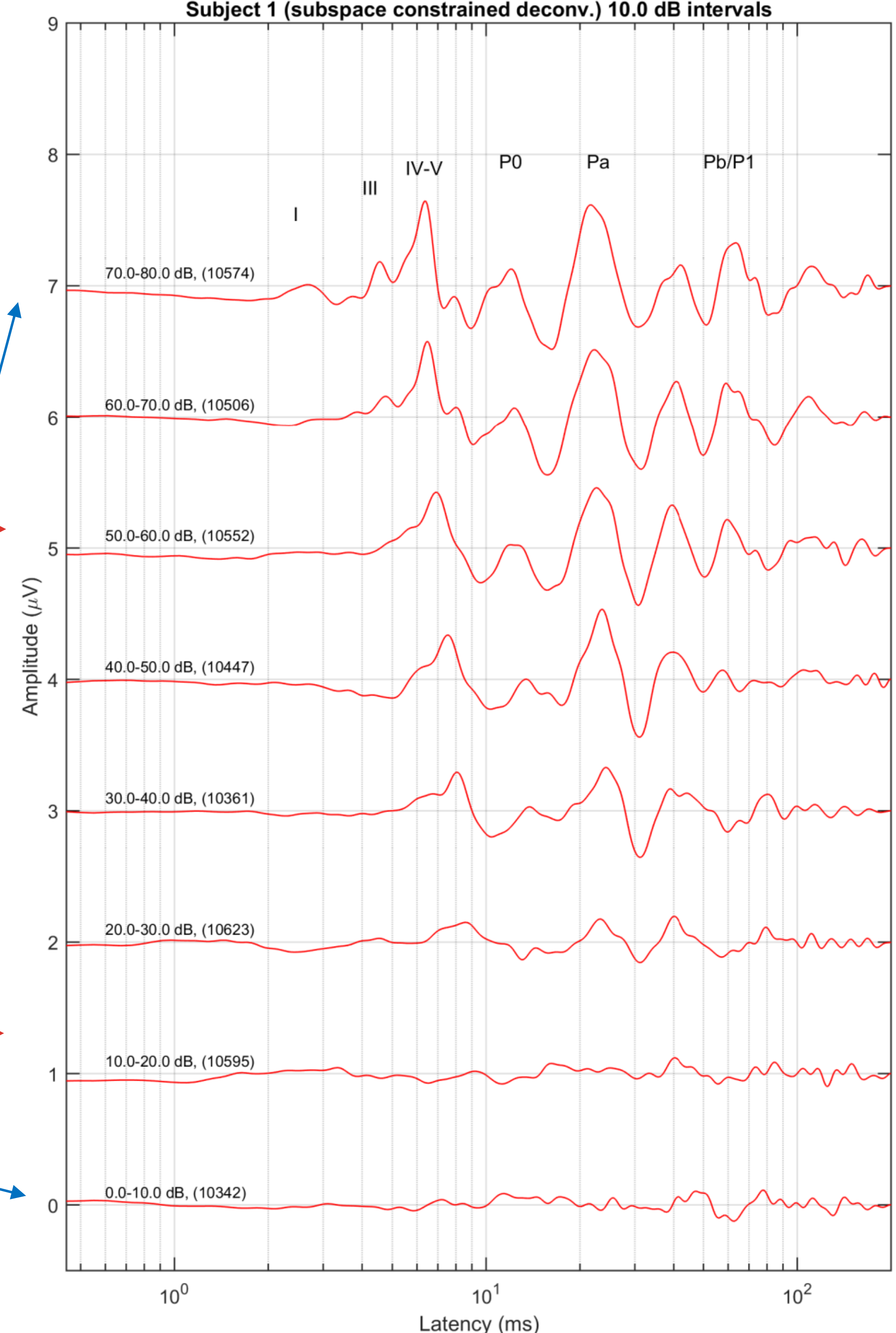


- AEPs of 200 ms @ ~~16,384 Hz~~ 40 functions/decade → ~~$J = 3{,}277$ samples~~ $J_{red} = 91$ samples
- $K = 10$ classes → ~~$(S^{\mathrm{T}}S)_{(32{,}770 \times 32{,}770)}$~~ $(S_{red}^{\mathrm{T}}S_{red})_{(910 \times 910)}$
- Deconvolution takes ~~1065 s~~ 30 s
- For $K > 10$ *classes*, ~~*Out-of-memory!*~~ Deconvolution is now feasible

Research possibilities

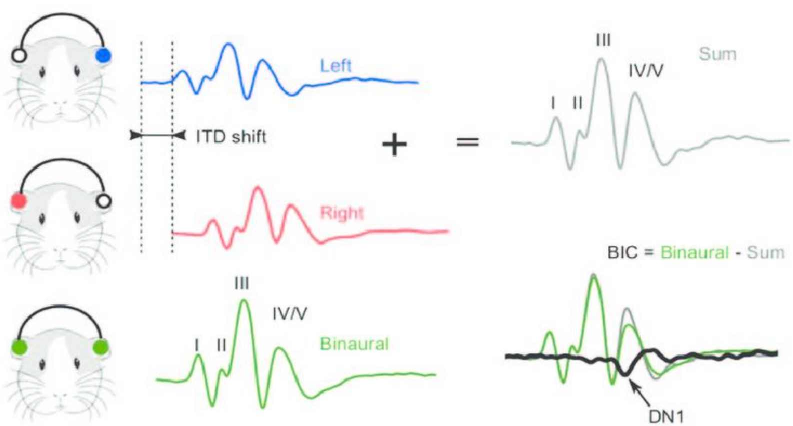
Threshold estimation





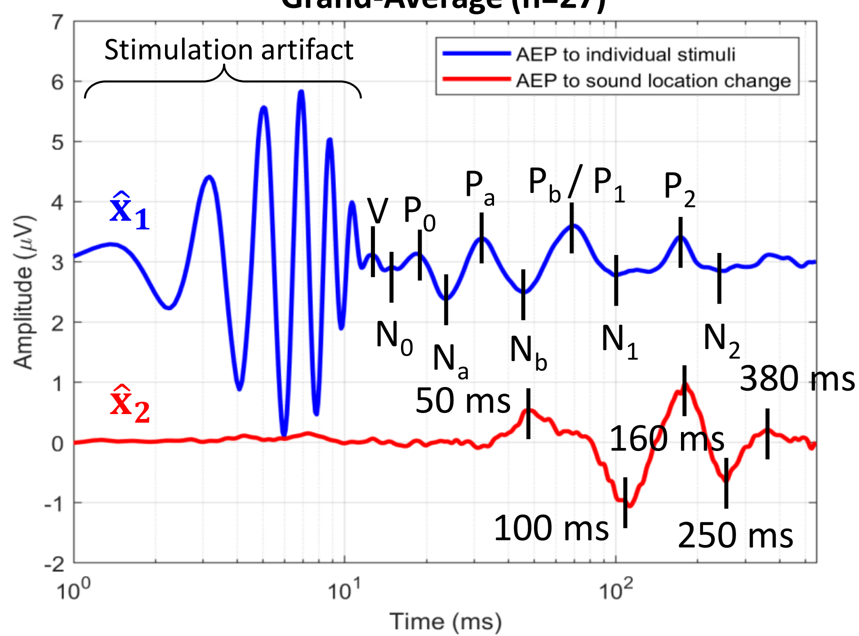
Binaural hearing

Binaural Interaction Component (BIC)

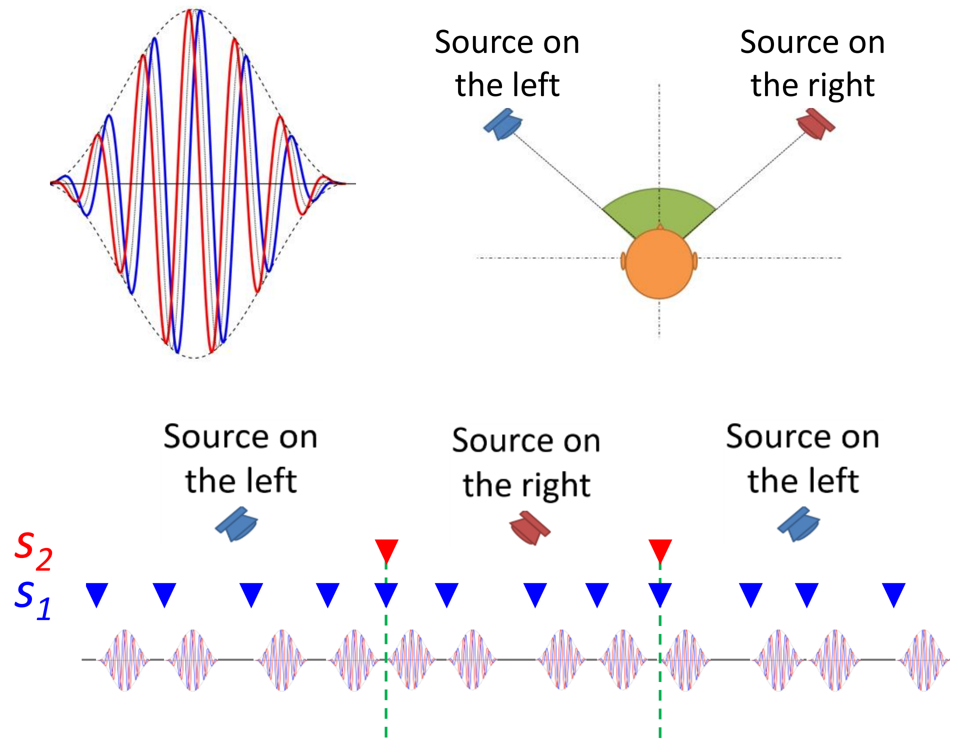


Ferber et al. (2016)

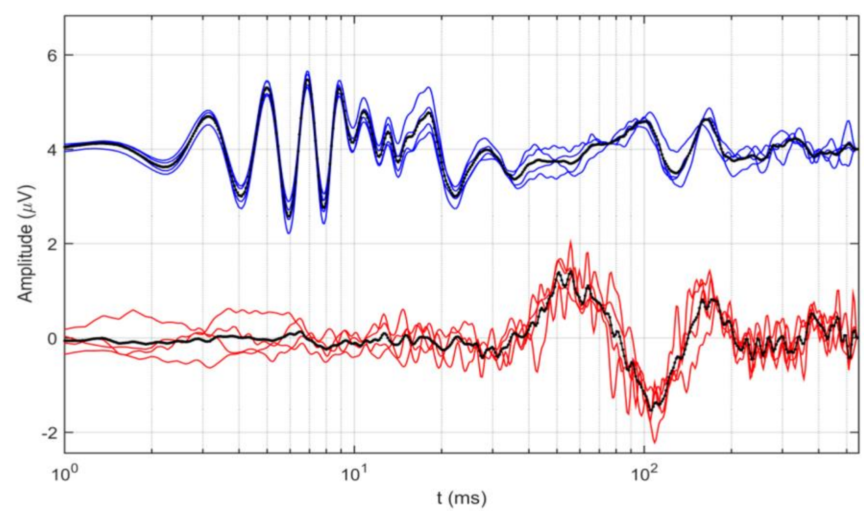
Grand-Average (n=27)



Deconvolution of multiple overlapping AEPs



Individual subject



Neural adaptation

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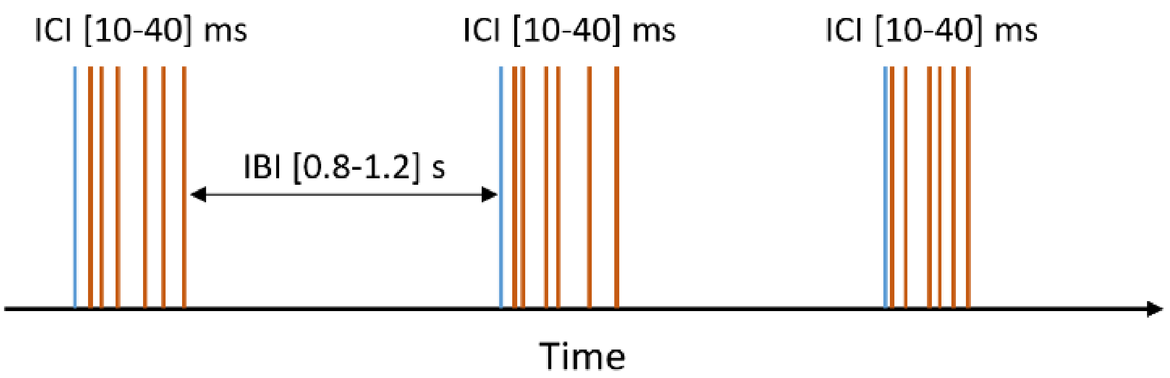

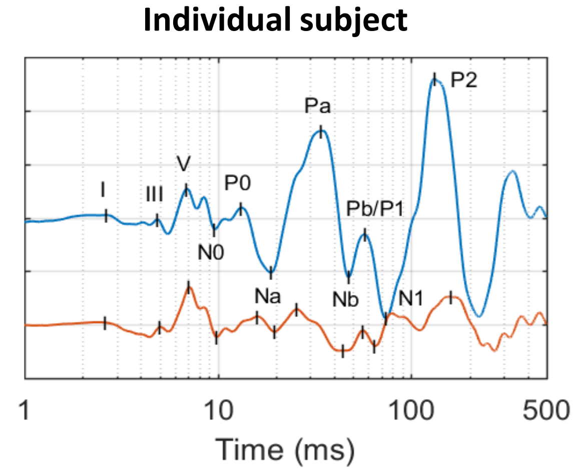

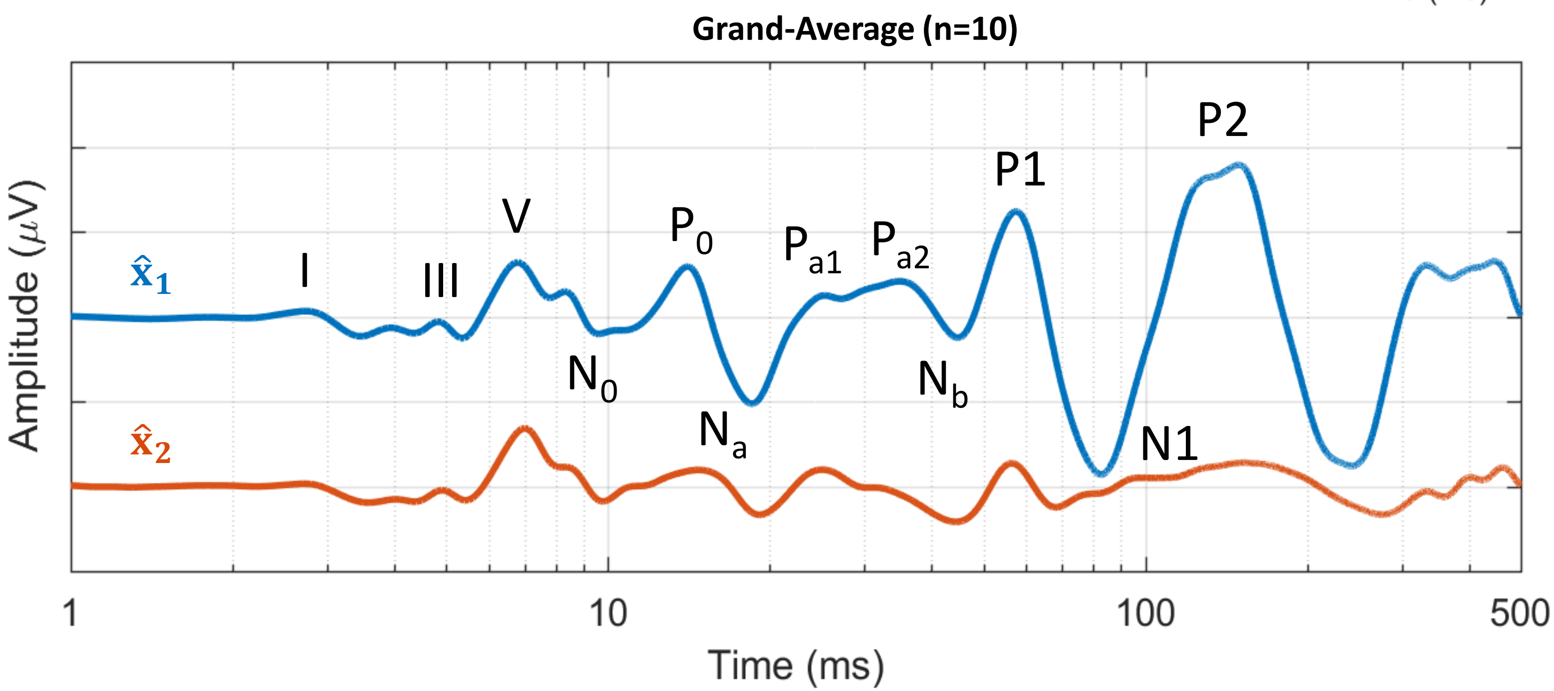

Speech processing

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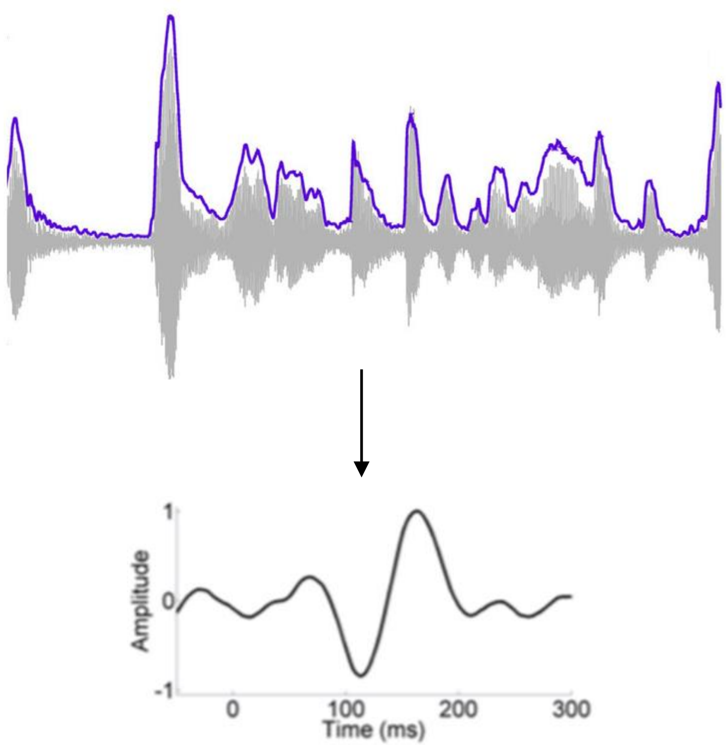
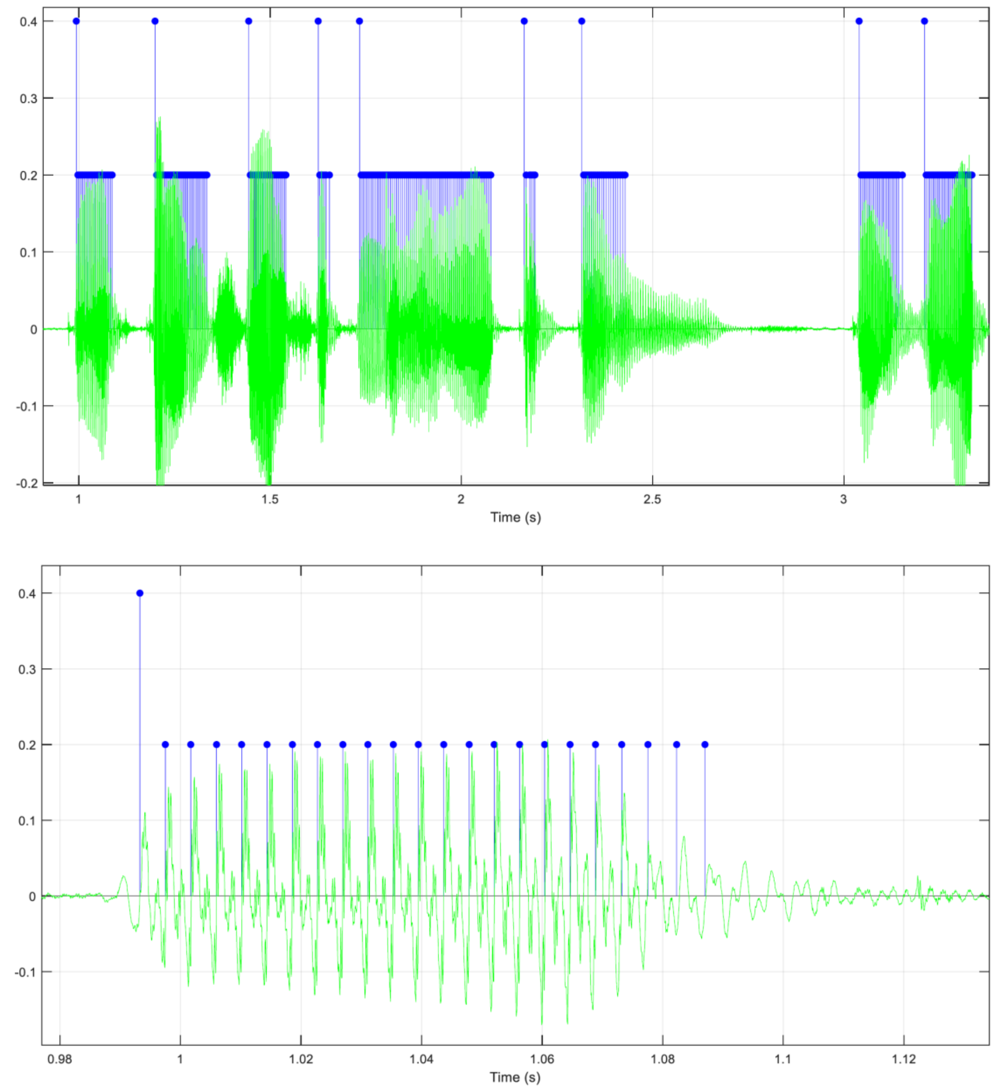
Temporal Response Function



Deconvolution of multiple overlapping AEPs



References

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Matrix deconvolution

Matrix-based formulation of the iterative randomized stimulation and averaging method for recording evoked potentials

Angel de la Torre,[1] Joaquin T. Valderrama,[2,a)] Jose C. Segura,[1] and Isaac M. Alvarez[1]
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[2]*National Acoustic Laboratories, Sydney, Australia*

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The iterative randomized stimulation and averaging (IRSA) method was proposed for recording evoked potentials when the individual responses are overlapped. The main inconvenience of IRSA is its computational cost, associated with a large number of iterations required for recovering the evoked potentials and the computation required for each iteration [involving the whole electroencephalogram (EEG)]. This article proposes a matrix-based formulation of IRSA, which is mathematically equivalent and saves computational load (because each iteration involves just a segment with the length of the response, instead of the whole EEG). Additionally, it presents an analysis of convergence that demonstrates that IRSA converges to the least-squares (LS) deconvolution. Based on the convergence analysis, some optimizations for the IRSA algorithm are proposed. Experimental results (configured for obtaining the full-range auditory evoked potentials) show the mathematical equivalence of the different IRSA implementations and the LS-deconvolution and compare the respective computational costs of these implementations under different conditions. The proposed optimizations allow the practical use of IRSA for many clinical and research applications and provide a reduction of the computational cost, very important with respect to the conventional IRSA, and moderate with respect to the LS-deconvolution. MATLAB/Octave implementations of the different methods are provided as supplementary material. *© 2019 Acoustical Society of America.*
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[BLM] Pages: 4545–4556

Latency-dependent filtering and downsampling

JASA ARTICLE

Latency-dependent filtering and compact representation of the complete auditory pathway response

Angel de la Torre,[1,a)] Joaquin T. Valderrama,[2,b)] Jose C. Segura,[1,c)] and Isaac M. Alvarez[1,d)]
[1]*Department of Signal Theory, Telematics, and Communications, University of Granada, Granada, Spain*
[2]*National Acoustic Laboratories, Sydney, Australia*

ABSTRACT:
Auditory evoked potentials (AEPs) include the auditory brainstem response (ABR), middle latency response (MLR), and cortical auditory evoked potentials (CAEPs), each one covering a specific latency range and frequency band. For this reason, ABR, MLR, and CAEP are usually recorded separately using different protocols. This article proposes a procedure providing a latency-dependent filtering and down-sampling of the AEP responses. This way, each AEP component is appropriately filtered, according to its latency, and the complete auditory pathway response is conveniently represented (with the minimum number of samples, i.e., without unnecessary redundancies). The compact representation of the complete response facilitates a comprehensive analysis of the evoked potentials (keeping the natural continuity related to the neural activity transmission along the auditory pathway), which provides a new perspective in the design and analysis of AEP experiments. Additionally, the proposed compact representation reduces the storage or transmission requirements when large databases are manipulated for clinical or research purposes. The analysis of the AEP responses shows that a compact representation with 40 samples/decade (around 120 samples) is enough for accurately representing the response of the complete auditory pathway and provides appropriate latency-dependent filtering. MATLAB/Octave code implementing the proposed procedure is included in the supplementary materials. *© 2020 Acoustical Society of America.* https://doi.org/10.1121/10.0001673
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Deconvolution in a reduced representation space

JASA ARTICLE

Subspace-constrained deconvolution of auditory evoked potentials

Angel de la Torre,[1,a)] Joaquin T. Valderrama,[2,b)] Jose C. Segura,[1,a)] Isaac M. Alvarez,[1,a)] and Jesus Garcia-Miranda[3]
[1]*Department of Signal Theory, Telematics, and Communications, University of Granada, Granada, Spain*
[2]*National Acoustic Laboratories, Sydney, Australia*
[3]*Department of Algebra, University of Granada, Granada, Spain*

ABSTRACT:
Auditory evoked potentials can be estimated by synchronous averaging when the responses to the individual stimuli are not overlapped. However, when the response duration exceeds the inter-stimulus interval, a deconvolution procedure is necessary to obtain the transient response. The iterative randomized stimulation and averaging and the equivalent randomized stimulation with least squares deconvolution have been proven to be flexible and efficient methods for deconvolving the evoked potentials, with minimum restrictions in the design of stimulation sequences. Recently, a latency-dependent filtering and down-sampling (LDFDS) methodology was proposed for optimal filtering and dimensionality reduction, which is particularly useful when the evoked potentials involve the complete auditory pathway response (i.e., from the cochlea to the auditory cortex). In this case, the number of samples required to accurately represent the evoked potentials can be reduced from several thousand (with conventional sampling) to around 120. In this article, we propose to perform the deconvolution in the reduced representation space defined by LDFDS and present the mathematical foundation of the subspace-constrained deconvolution. Under the assumption that the evoked response is appropriately represented in the reduced representation space, the proposed deconvolution provides an optimal least squares estimation of the evoked response. Additionally, the dimensionality reduction provides a substantial reduction of the computational cost associated with the deconvolution. MATLAB/Octave code implementing the proposed procedures is included as supplementary material.
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Multi-response deconvolution

Under review

To take-home

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- **Matrix Deconvolution** enables evoked potentials to be recorded at fast rates, increasing flexibility in experimental design.
- The compact representation provided by **latency-dependent filtering and downsampling** (LDFDS) facilitates (1) a comprehensive representation of evoked potentials along the auditory pathway, and (2) an important dimensionality reduction.
- Performing **deconvolution in the reduced space** defined by LDFDS significantly reduces computational load.
- **Multi-response deconvolution** is appropriate to model multiple neurophysiological processes evoked by complex stimuli.
- MATLAB / Octave toolkits with functions and simulations are available to help understand and use these methodologies.

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