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Simultaneous deconvolution of multiple auditory evoked potentials in a reduced representation space

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From simple equations to multi-response deconvolution

Simple equation with 1 unknown

$$y = a\,x \qquad \text{unknown:} \quad x$$

System of 2 equations with 2 unknowns

$$\begin{cases} y_1 & = & a_{1,1}\,x_1 + a_{1,2}\,x_2 \\ y_2 & = & a_{2,1}\,x_1 + a_{2,2}\,x_2 \end{cases}$$

$$\text{unknowns:} \quad x_1, x_2$$

Matrix formulation of the system of equations

$$\begin{cases} y_1 & = & a_{1,1}\, x_1 + a_{1,2}\, x_2 \\ y_2 & = & a_{2,1}\, x_1 + a_{2,2}\, x_2 \end{cases}$$

unknowns: x_1, x_2

$$\mathbf{y} = A\,\mathbf{x}$$

$$A^{-1}\,\mathbf{y} = A^{-1}\,A\,\mathbf{x} = I\,\mathbf{x} = \mathbf{x}$$

$$\mathbf{x} = A^{-1}\,\mathbf{y}$$

Overdetermined system of equations
(no solution except if equations are redundant)

$$\begin{cases} y_1 & = & a_{1,1}\, x_1 + a_{1,2}\, x_2 \\ y_2 & = & a_{2,1}\, x_1 + a_{2,2}\, x_2 \\ y_3 & = & a_{3,1}\, x_1 + a_{3,2}\, x_2 \\ y_4 & = & a_{4,1}\, x_1 + a_{4,2}\, x_2 \\ y_5 & = & a_{5,1}\, x_1 + a_{5,2}\, x_2 \\ \ldots & & \\ y_N & = & a_{N,1}\, x_1 + a_{N,2}\, x_2 \end{cases}$$

unknowns: x_1, x_2

Overdetermined system of equations (makes sense in noise)

$$\begin{cases} y_1 & = & a_{1,1}\,x_1 + a_{1,2}\,x_2 + n_1 \\ y_2 & = & a_{2,1}\,x_1 + a_{2,2}\,x_2 + n_2 \\ y_3 & = & a_{3,1}\,x_1 + a_{3,2}\,x_2 + n_3 \\ y_4 & = & a_{4,1}\,x_1 + a_{4,2}\,x_2 + n_4 \\ y_5 & = & a_{5,1}\,x_1 + a_{5,2}\,x_2 + n_5 \\ \dots & & \\ y_N & = & a_{N,1}\,x_1 + a_{N,2}\,x_2 + n_N \end{cases}$$

unknowns: x_1, x_2

Overdetermined system of equations in noise (all equations are useful): the LS solution

$$\mathbf{y} = A\,\mathbf{x} + \mathbf{n}$$

$$\nexists \; A^{-1}$$

$$\hat{\mathbf{x}} = \left(A^T A\right)^{-1} A^T \mathbf{y}$$

$$\|\mathbf{y} - A\mathbf{x}\|^2 \quad \text{minimum residual}$$

Convolution as a system of equations

The response samples are the unknowns; The EEG is the observation; the stimulation sequence provides the coefficients

$$y(n) = s(n) * x(n) + n_0(n)$$

Matrix representation

$$\mathbf{y} = S\mathbf{x} + \mathbf{n}_0$$

LS solution

$$\hat{\mathbf{x}} = \left(S^T S\right)^{-1} S^T \mathbf{y}$$

Autocorrelation of stimulation sequence / synchronous average

$$\hat{\mathbf{x}} = R_s^{-1} \mathbf{z}_0$$

Practical problems in deconvolution:

- Invertibility of the matrix
- Is the matrix singular (null eigenvalues)?
- Is quasi-singular (near null eigenvalues)?
- Are there negative eigenvalues? (!!!!)
- Periodic vs random sequences (resonances)
- Amplification of noise
- Computational cost: $O(N^{2.4})$

$$R_s \qquad (J \times J) \text{ matrix}$$

$$J \text{ large} \Rightarrow \text{PROBLEMS!}$$

Solution: subspace constrained deconv.

Reduction of the dimensionality with orthonormal projector

$$J \;\rightarrow\; J_r \qquad \mathbf{x}_r = V_r\,\mathbf{x} \qquad \mathbf{x} = V_r^T\,\mathbf{x}_r$$

$$\mathbf{y} = S\,V_r^T\,\mathbf{x}_r + \mathbf{n}_0$$

$$\begin{aligned}
\hat{\mathbf{x}}_r &= \left((S\,V_r^T)^T(S\,V_r^T)\right)^{-1}(S\,V_r^T)^T\,\mathbf{y} = \\
&= \left(V_r\,S^T\,S\,V_r^T\right)^{-1}V_r\,S^T\,\mathbf{y} = \\
&= \left(V_r\,R_s\,V_r^T\right)^{-1}V_r\,\mathbf{z}_0 = R_{sr}^{-1}\,\mathbf{z}_{0r}
\end{aligned}$$

Advantages of dimensionality reduction:

- **Reduction of noise**
- **Reduction of computational cost**
- **Reduction of problems with invertibility (condition number reduced)**
- **Diagnose of matrix (eigenvalues) easier**

$$\hat{\mathbf{x}}_r = R_{sr}^{-1}\,\mathbf{z}_{0r}$$

$$R_{sr} \qquad (J_r \times J_r)\ \text{matrix}$$

$$\hat{\mathbf{x}} = V_r^T\,\hat{\mathbf{x}}_r$$

How can dimensionality be reduced?

- **Example:**
  - **10 oscillations/s requires 20 samples/s**
  - **10 oscillations/decade requires 20 samples/dec.**
- **Conventional representation ABR/MLR/CAEP:**
  - **1 second (CAEP) at 10 kHz (ABR): J = 10.000 samples**
- **LDFDS**
  - **3 decades (1 ms to 1 s)**
  - **40 samples/dec. (3 to 5 oscillations/dec. in ABR/MLR/CAEP)**
  - **J_r = 120 samples**

R_s is a (10.000 x 10.000) matrix with 10.000 eigenvalues

R_{sr} is a (120 x 120) matrix with 120 eigenvalues

Multi-response deconvolution

Convolution model with 2 categories of stimulus (2 stimulation sequences, 2 responses)

$$y(n) = s_1(n) * x_1(n) + s_2(n) * x_2(n) + n_0(n)$$

$$\mathbf{y} = S_1\mathbf{x}_1 + S_2\mathbf{x}_2 + \mathbf{n}_0$$

$$\mathbf{y} = [S_1 S_2] \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{bmatrix} + \mathbf{n}_0 = S_{all}\mathbf{x}_{all} + \mathbf{n}_0$$

LS solution for the multi-response problem:

$$\hat{\mathbf{x}}_{all} = \left[S_{all}^T S_{all}\right]^{-1} S_{all}^T \, \mathbf{y}$$

$$\begin{bmatrix} \hat{\mathbf{x}}_1 \\ \hat{\mathbf{x}}_2 \end{bmatrix} = \left(\begin{bmatrix} S_1^T \\ S_2^T \end{bmatrix} [S_1 S_2] \right)^{-1} \begin{bmatrix} S_1^T \\ S_2^T \end{bmatrix} \mathbf{y}$$

$$\begin{bmatrix} \hat{\mathbf{x}}_1 \\ \hat{\mathbf{x}}_2 \end{bmatrix} = \begin{bmatrix} R_{s\,11} & R_{s\,12} \\ R_{s\,21} & R_{s\,22} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{z}_{01} \\ \mathbf{z}_{02} \end{bmatrix}$$

$$\hat{\mathbf{x}}_{all} = R_{s\,all}^{-1} \, \mathbf{z}_{0all}$$

LS solution for the multi-response: a *J x M* dimension problem

$$\hat{\mathbf{x}}_{all} = R_{s\,all}^{-1}\,\mathbf{z}_{0all}$$

$R_{s\,all}$ $\quad ((J \times M) \times (J \times M))$ matrix

Multi-response deconvolution constrained to the reduced subspace

Similar to multi-response deconvolution but matrix inversion performed in a reduced representation space:

$$\begin{bmatrix} \hat{\mathbf{x}}_{r1} \\ \hat{\mathbf{x}}_{r2} \end{bmatrix} = \begin{bmatrix} R_{sr\,11} & R_{sr\,12} \\ R_{sr\,21} & R_{sr\,22} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{z}_{0r1} \\ \mathbf{z}_{0r2} \end{bmatrix}$$

$$\hat{\mathbf{x}}_{r\,all} = R_{sr\,all}^{-1}\,\mathbf{z}_{0r\,all}$$

$$R_{sr\,all} \qquad ((J_r \times M) \times (J_r \times M)) \ \text{matrix}$$

Summary

Dimensionality of the different deconvolution problems

Conventional deconvolution	R_s	$(J \times J)$ matrix
Subspace deconvolution	R_{sr}	$(J_r \times J_r)$ matrix
Multi-response deconv.	$R_{s\,all}$	$((J \times M) \times (J \times M))$ matrix
Subspace multi-resp. deconv.	$R_{sr\,all}$	$((J_r \times M) \times (J_r \times M))$ matrix

Summary

Dimensionality of the different deconvolution problems

$J = 10.000; J_r = 120; M = 10$

Conventional deconvolution	R_s	(10.000×10.000) matrix
Subspace deconvolution	R_{sr}	(120×120) matrix
Multi-response deconv.	$R_{s\,all}$	$(100.000 \times 100.000$ matrix
Subspace multi-resp. deconv.	$R_{sr\,all}$	(1.200×1.200) matrix

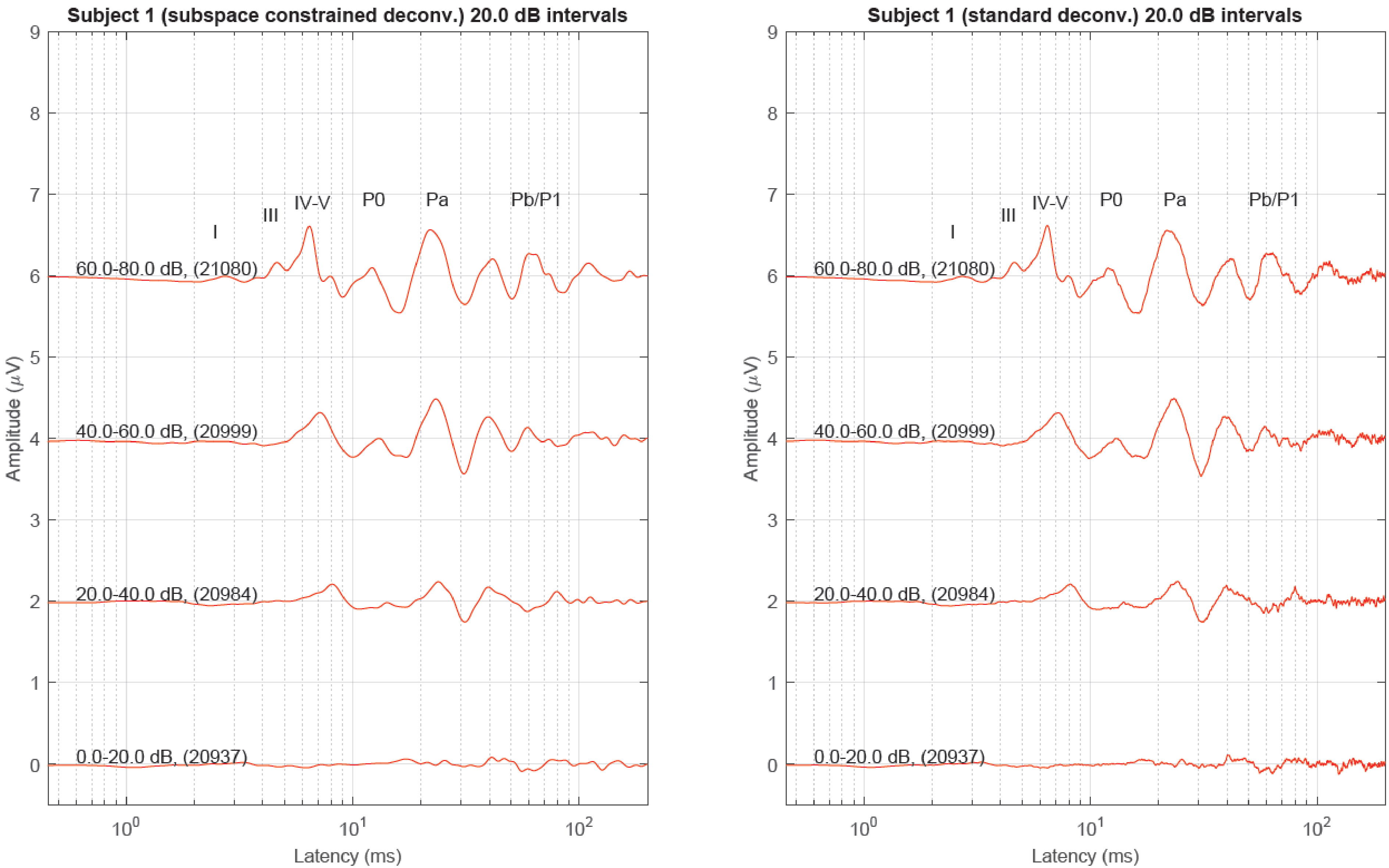
Experiments

- **Stimulation**
  - Clicks, random ISI (15-30 ms) and level (0-80 dB)
  - 31.5 minutes (84.000 stimuli)
  - Multi-response based on stimulation level
- **Recording**
  - Biosemi recording system sampled at 16384 Hz
  - Bandwidth 20-3300 Hz (ABR/MLR)
  - Response length: 200 ms (J=3277 samples)
- **Dimensionality reduction: 40 samples/dec.**
  - Reduced representation J_r=91 samples

- **Multi-response deconvolution**
  - **Categorization based on stimulation level**
  - **M categories in 80 dB range:**
    - **M=2: 2 categories of 40 dB (0-40 dB and 40-80 dB)**
    - **M=4: 4 categories of 20 dB**
    - **M=16: 16 categories of 5 dB**
  - **Categorization and deconvolution off-line (after recording)**
- **Deconvolution in complete and reduced representation space (transformed to conventional representation)**
- **Evaluation: noise and computational cost**

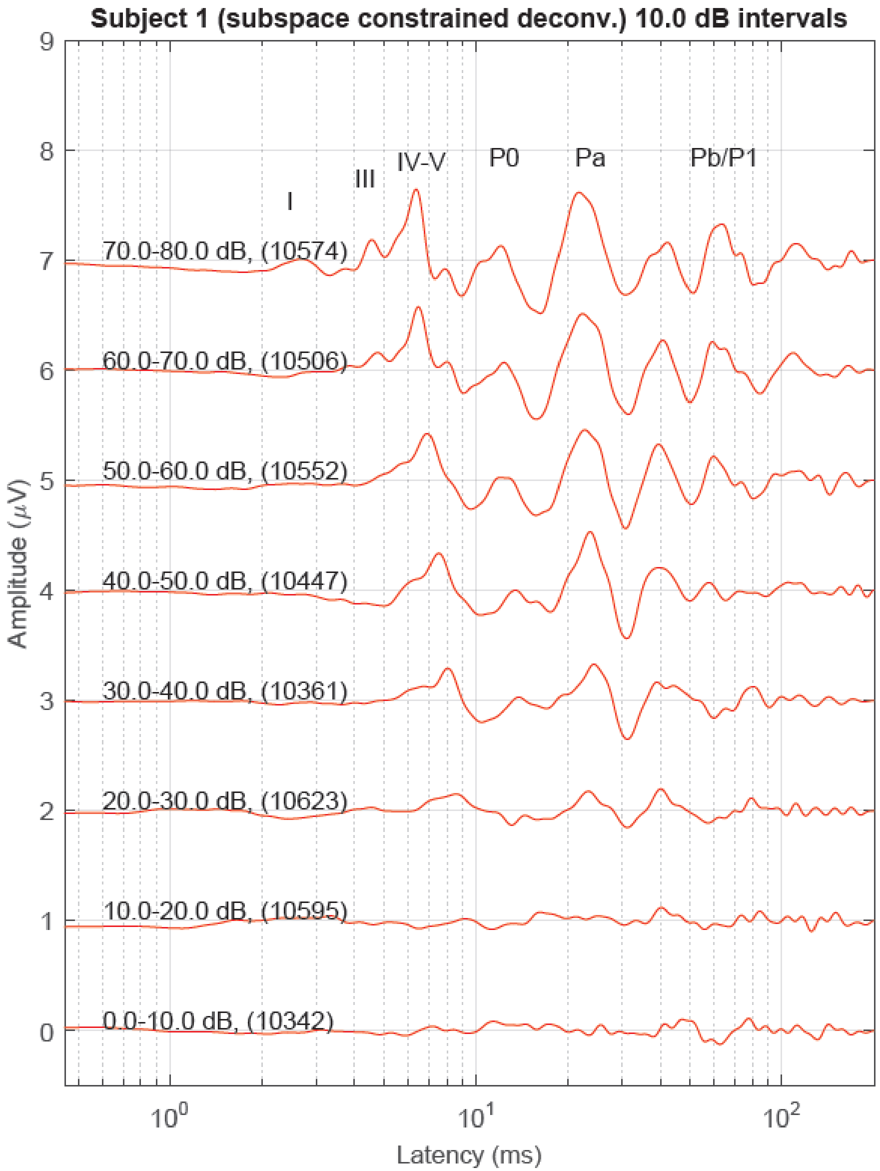
4 categories of 20 dB: reduction from 13108 to 364 dimensions

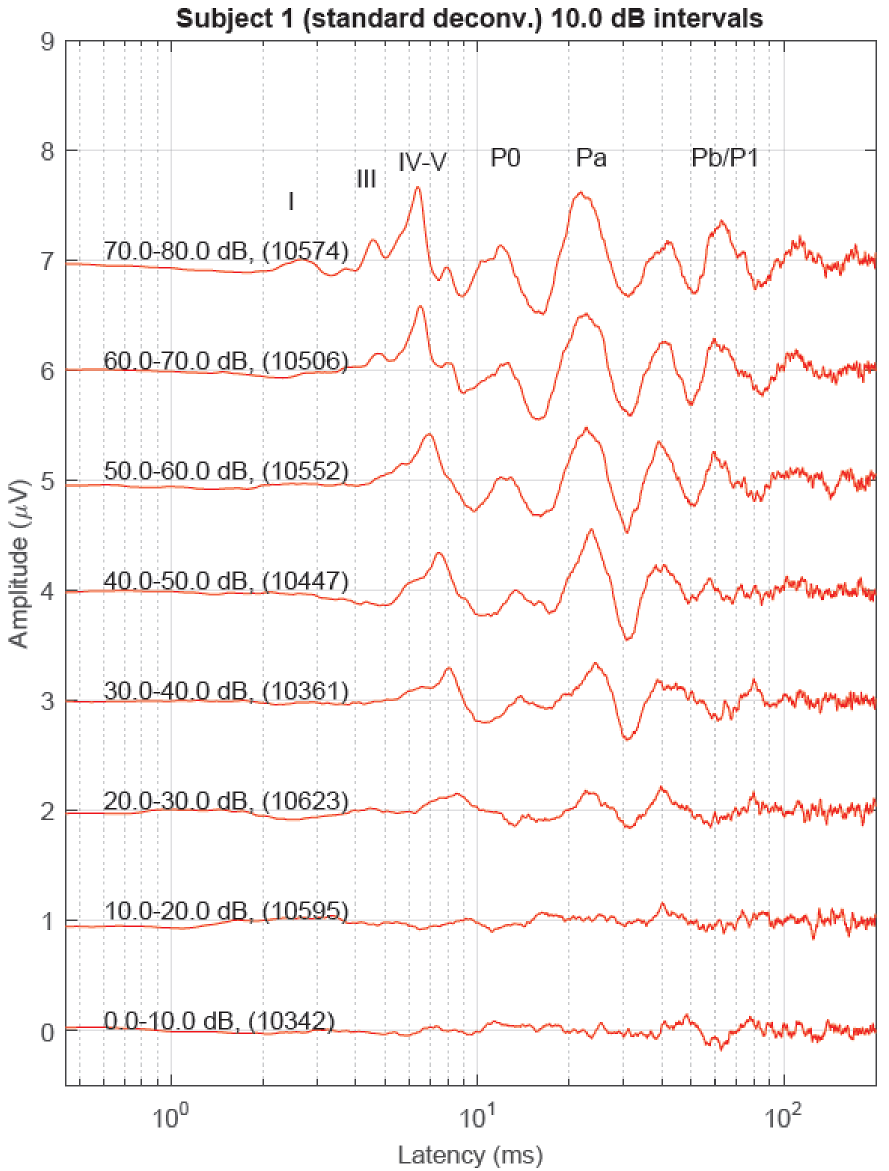
Computational cost: reduction from 28,4 to 10,7 seconds



8 categories of 10 dB: reduction from 26216 to 728 dimensions

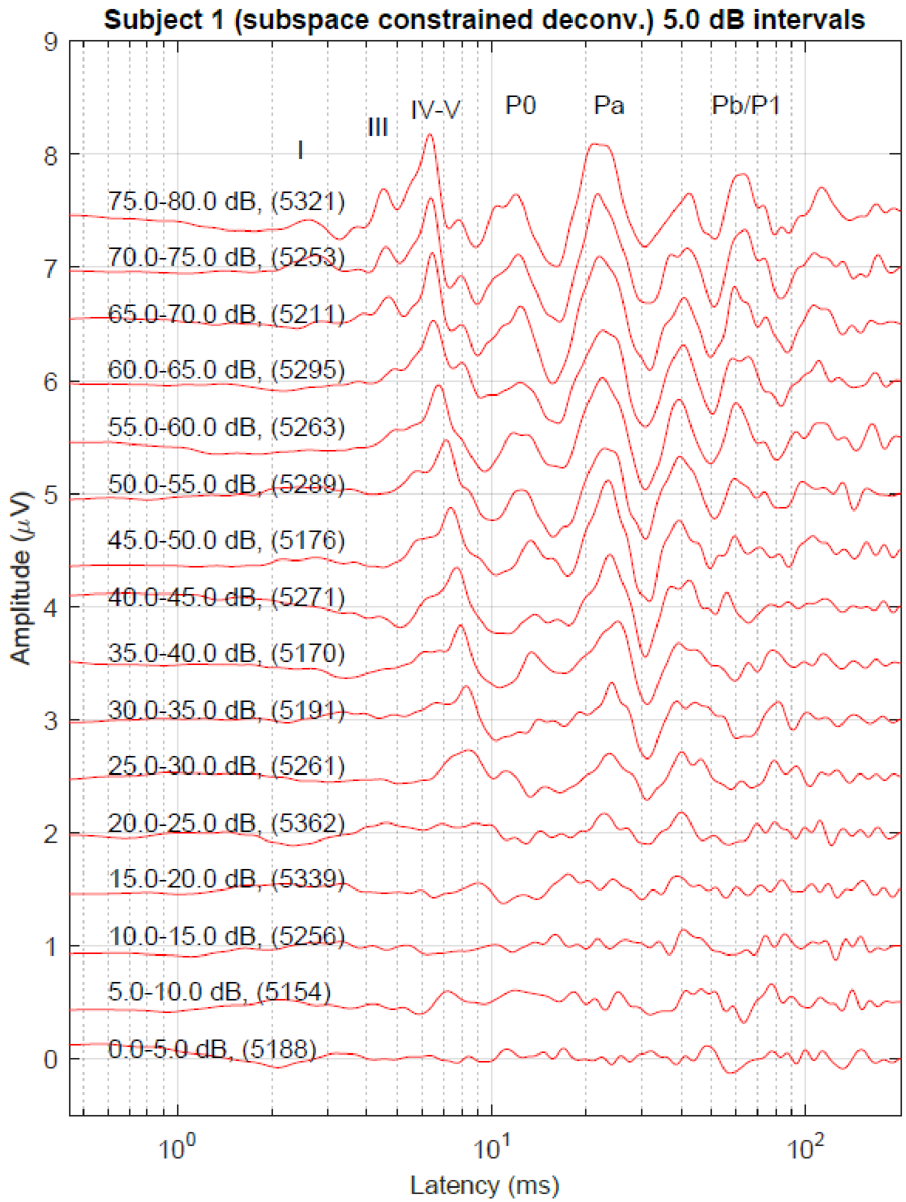
Computational cost: reduction from 304,4 to 22,8 seconds





16 categories of 5 dB: reduction from 52432 to 1456 dimens.

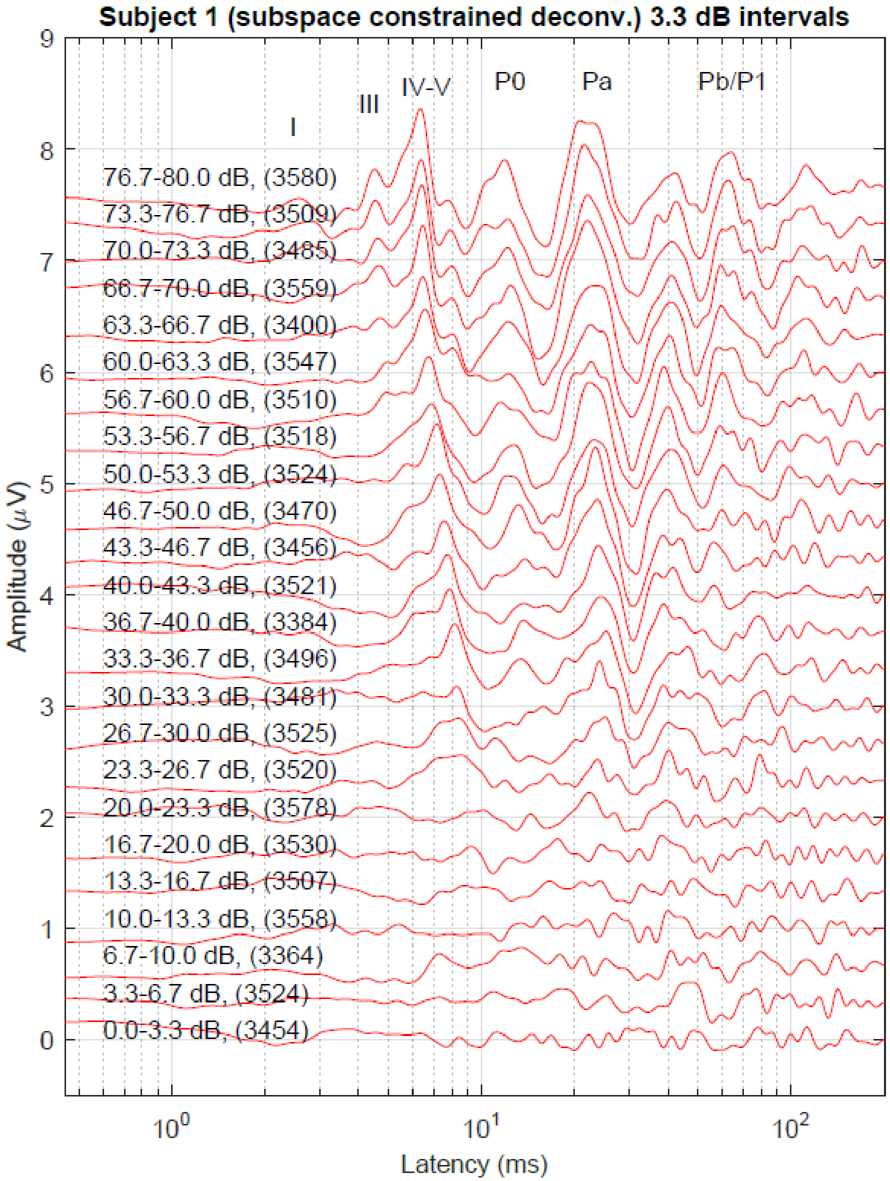
Computational cost: reduction from XXXX to 56,9 seconds

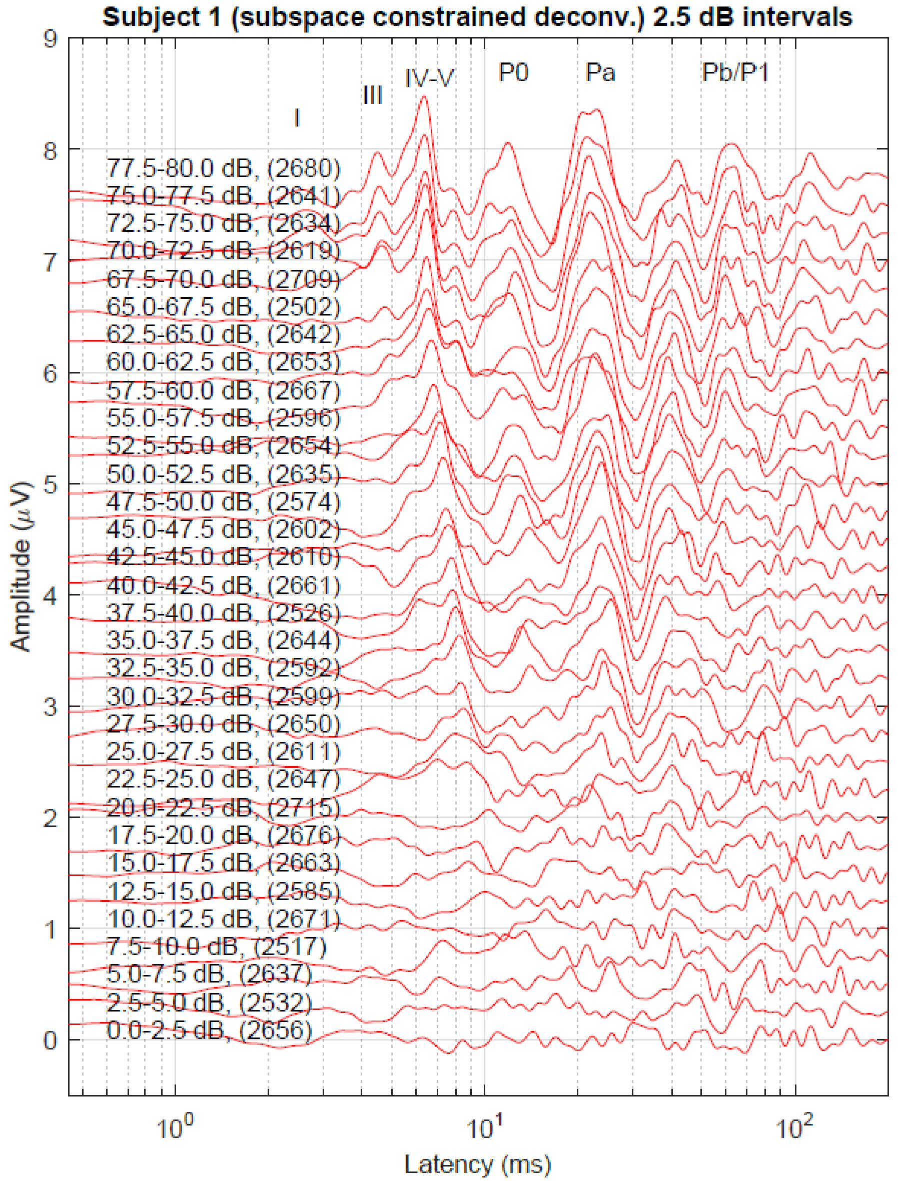


MEMORY OVERFLOW IN THE COMPLETE REPRESENTATION SPACE

24 categories of 3.3 dB and 32 categories of 2.5 dB (subspace)

Computational cost: 97,7 and 135,7 seconds

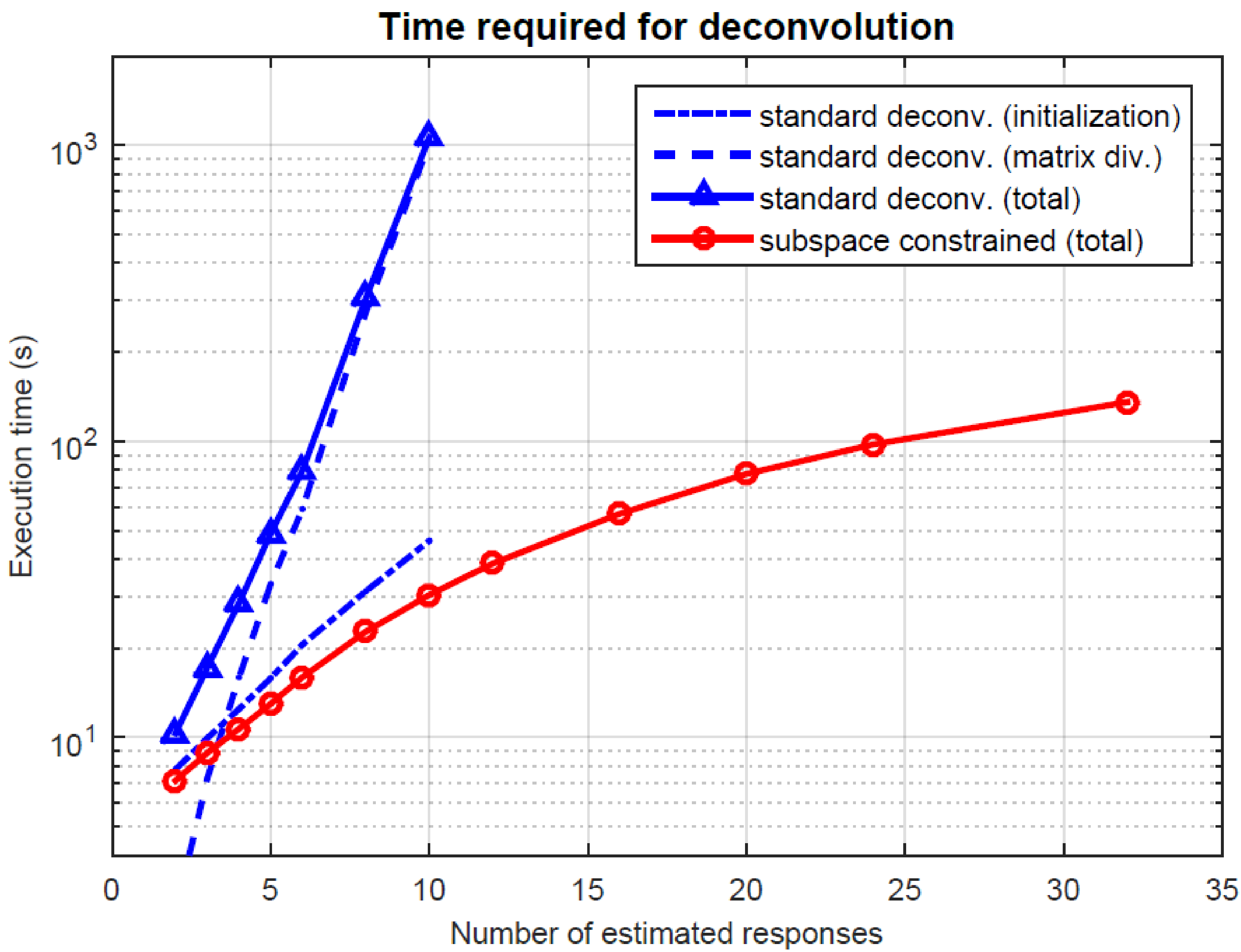




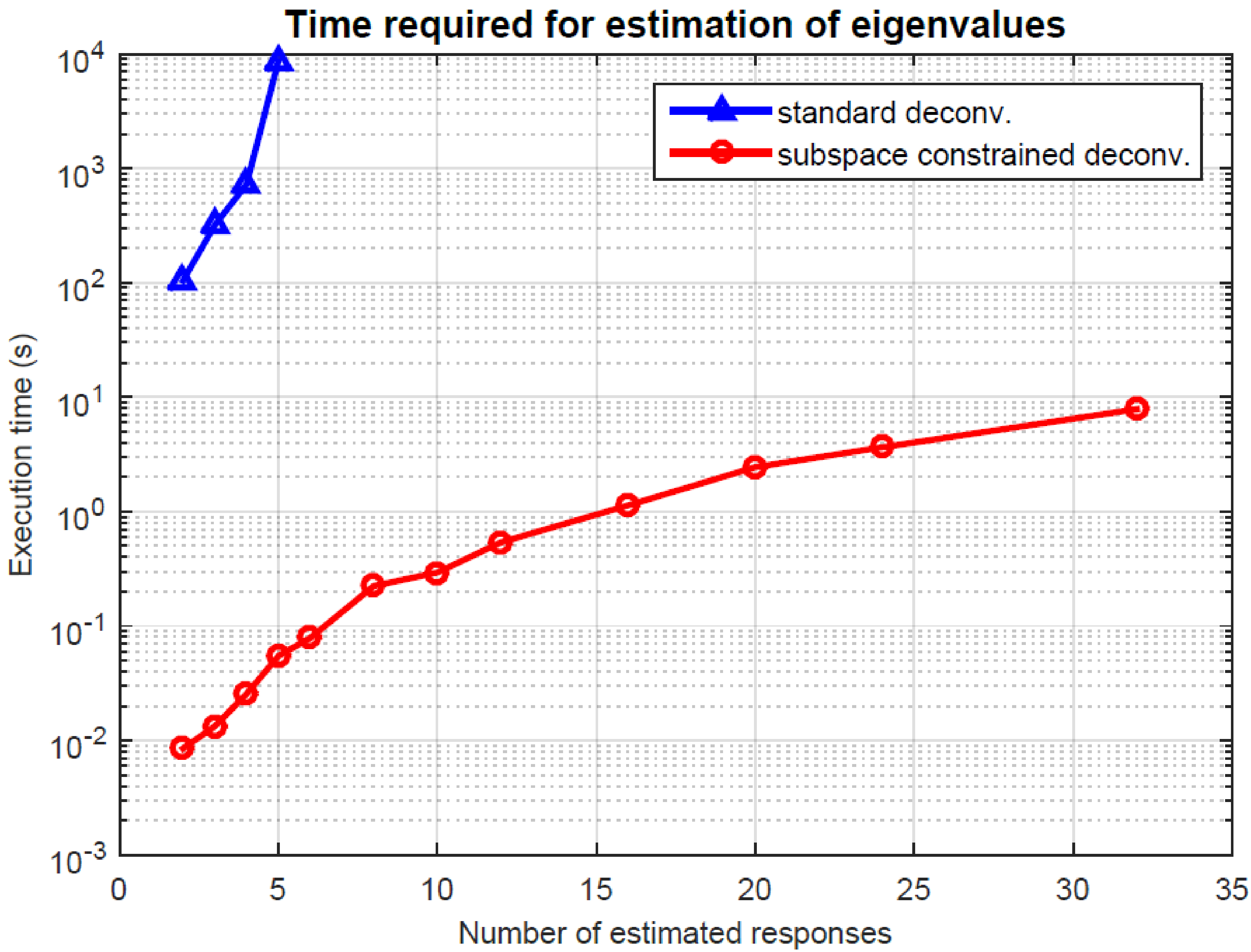
Computational load comparison

Experiment		Complete Representation Space						Subspace-Constrained Deconvolution					
Numb.	Interval	Numb.	execution time (seconds)				Cond.	Numb.	execution time (seconds)				Cond.
categ.	(dB)	dimen.	init.	deconv.	total	eigenval.	number	dimen.	init.	deconv.	total	eigenval.	number
2	40.0	6554	7.80	2.54	10.34	104.05	230.0	182	7.15	0.0004	7.15	0.0085	220.7
3	26.7	9831	9.95	7.28	17.23	318.37	246.8	273	8.87	0.0008	8.87	0.0133	220.7
4	20.0	13108	12.48	15.97	28.45	723.91	241.0	364	10.70	0.0017	10.70	0.0255	220.7
5	16.0	16385	15.86	33.01	48.88	8642.96	256.2	455	12.99	0.0024	12.99	0.0547	220.8
6	13.3	19662	20.60	59.16	79.76	mem. overflow		546	15.98	0.0049	15.99	0.0784	220.8
8	10.0	26216	31.20	273.21	304.41	mem. overflow		728	22.78	0.0067	22.79	0.2239	220.8
10	8.0	32770	46.17	1018.04	1064.21	mem. overflow		910	30.29	0.0120	30.30	0.2933	220.9
12	6.7	39324	memory overflow			mem. overflow		1092	38.68	0.0218	38.70	0.5386	220.9
16	5.0	52432	memory overflow			mem. overflow		1456	56.90	0.0531	56.95	1.1333	221.1
20	4.0	65540	memory overflow			mem. overflow		1820	77.56	0.1038	77.67	2.4461	221.1
24	3.3	78648	memory overflow			mem. overflow		2184	97.57	0.1114	97.68	3.6501	221.1
32	2.5	104864	memory overflow			mem. overflow		2912	135.48	0.2279	135.71	7.8493	221.5

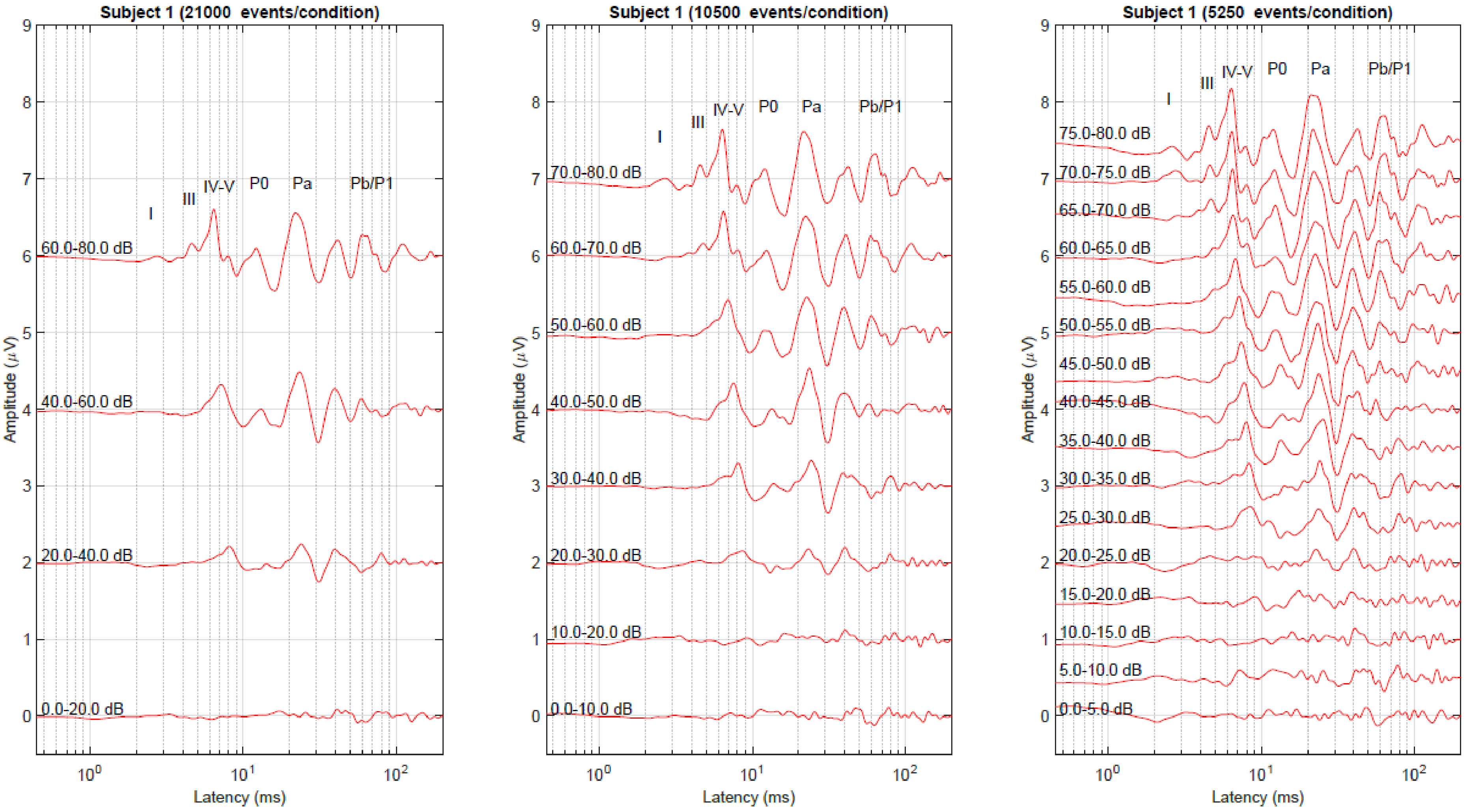
Computational load comparison



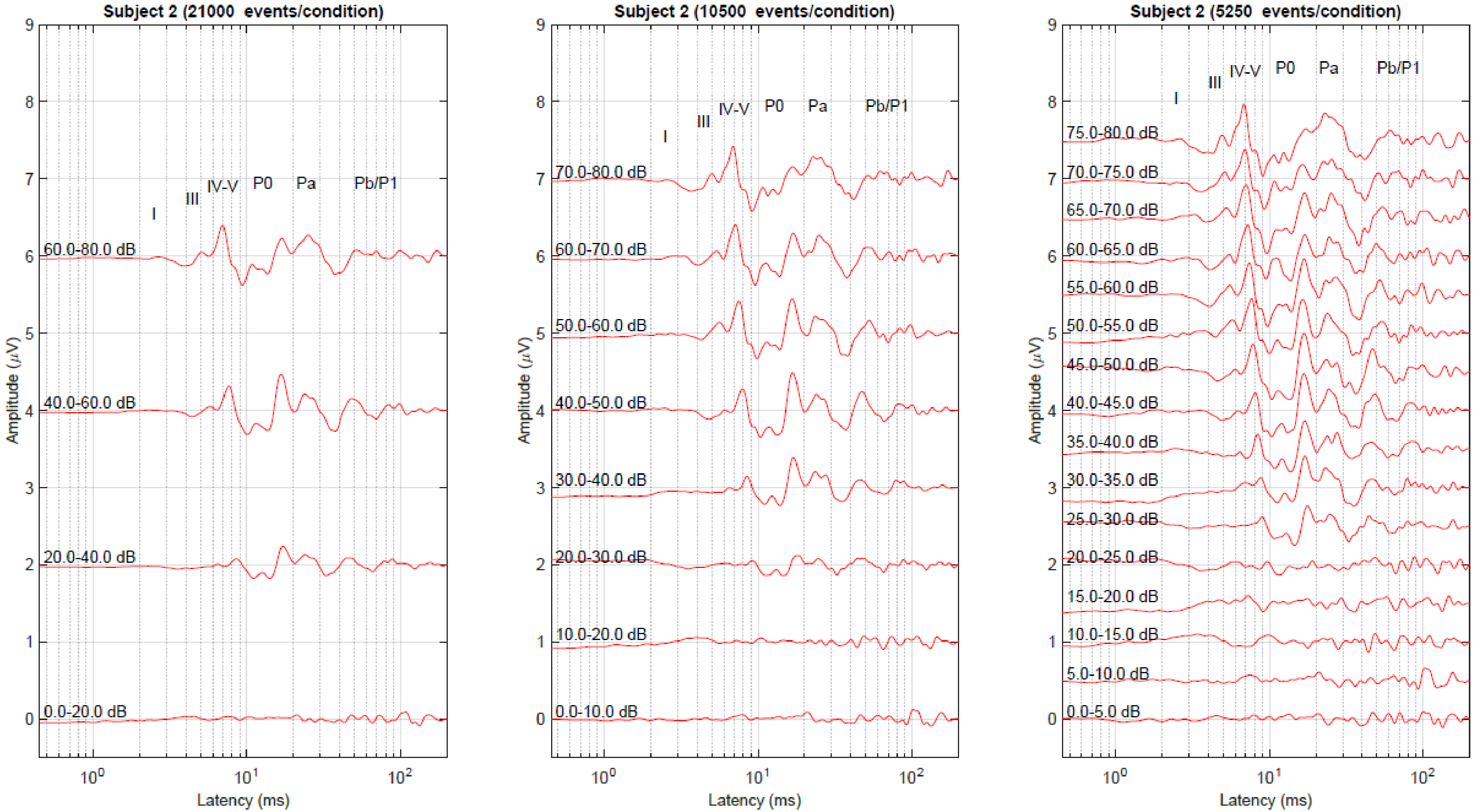
Computational load comparison



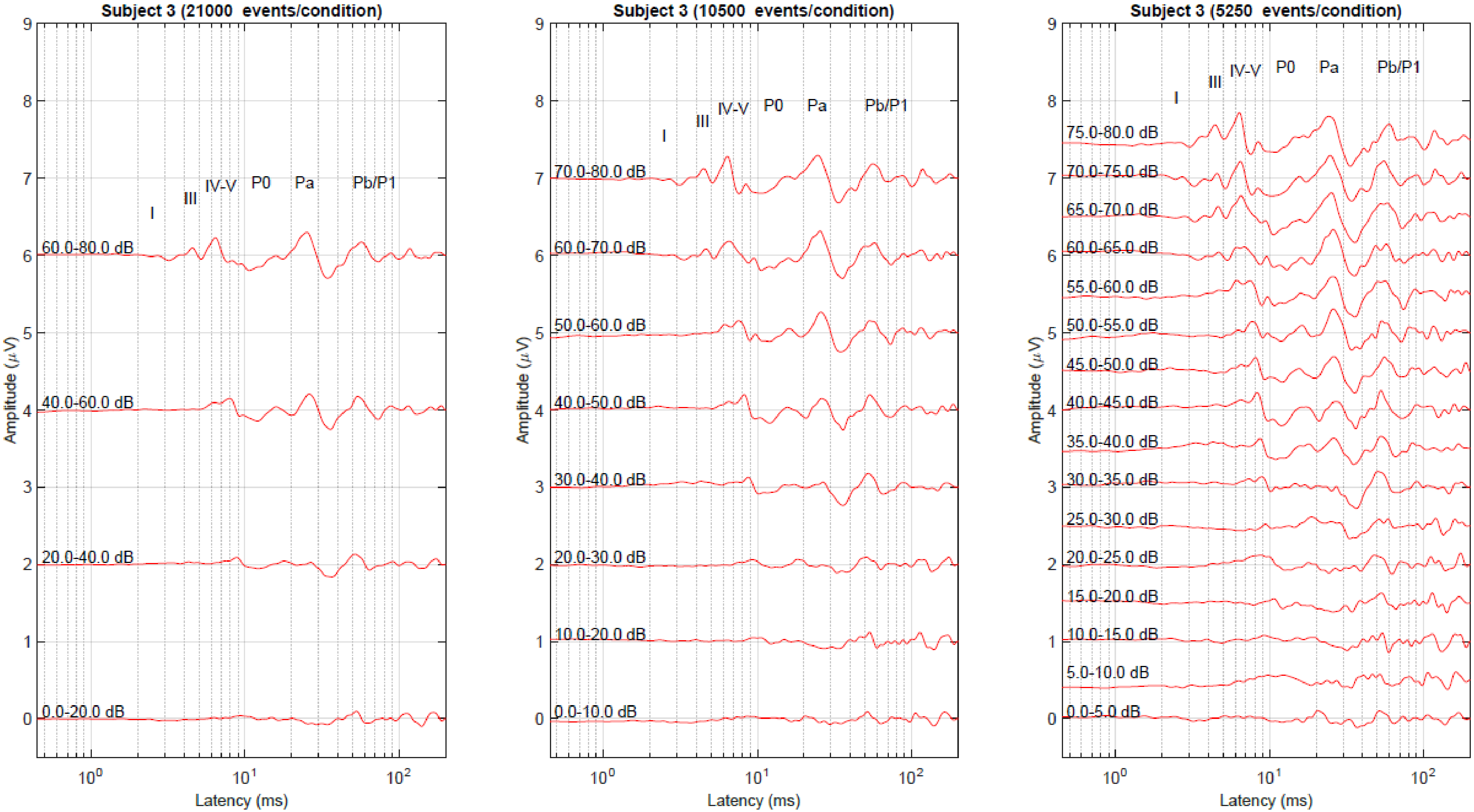
Individual responses: subject 1



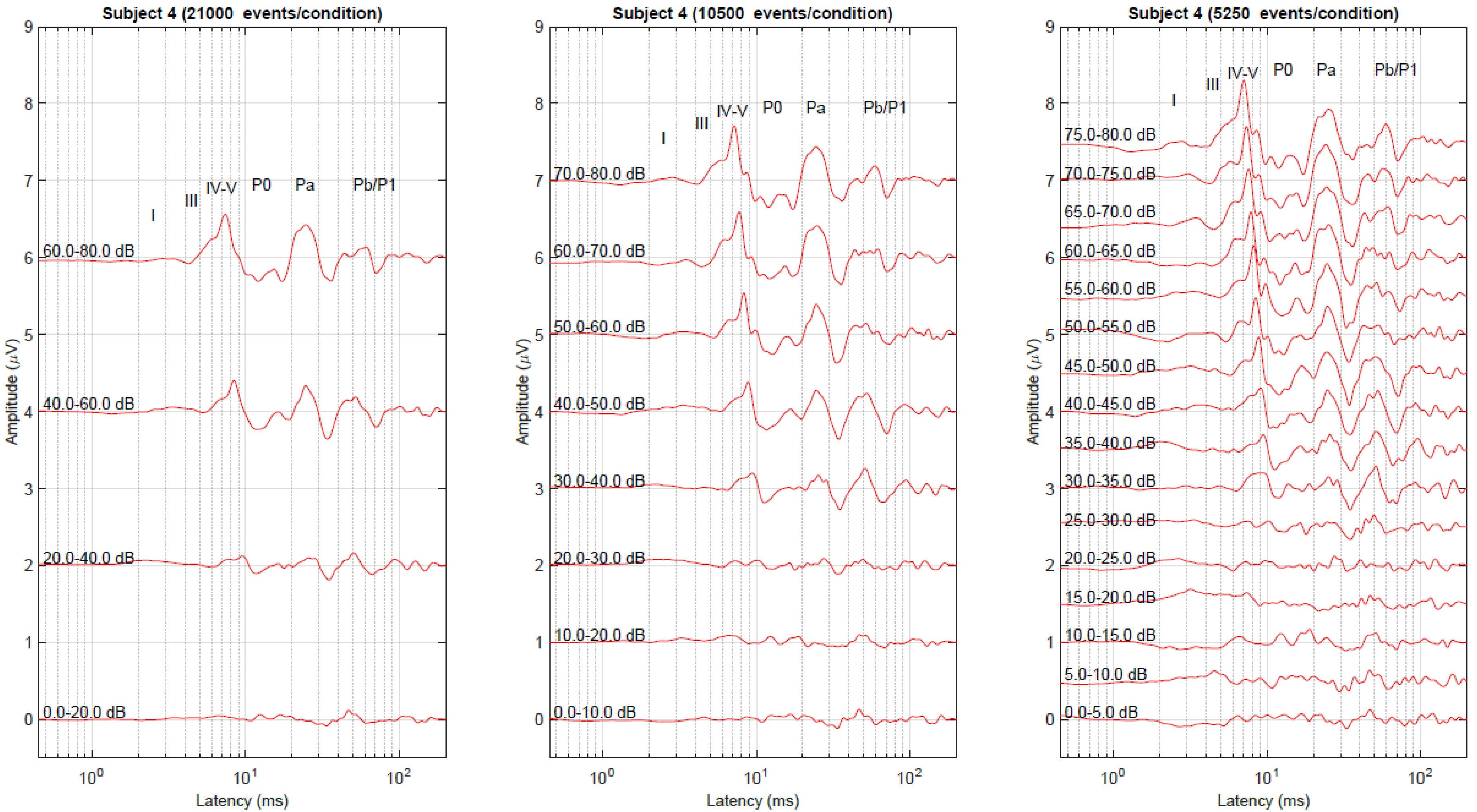
Individual responses: subject 2



Individual responses: subject 3

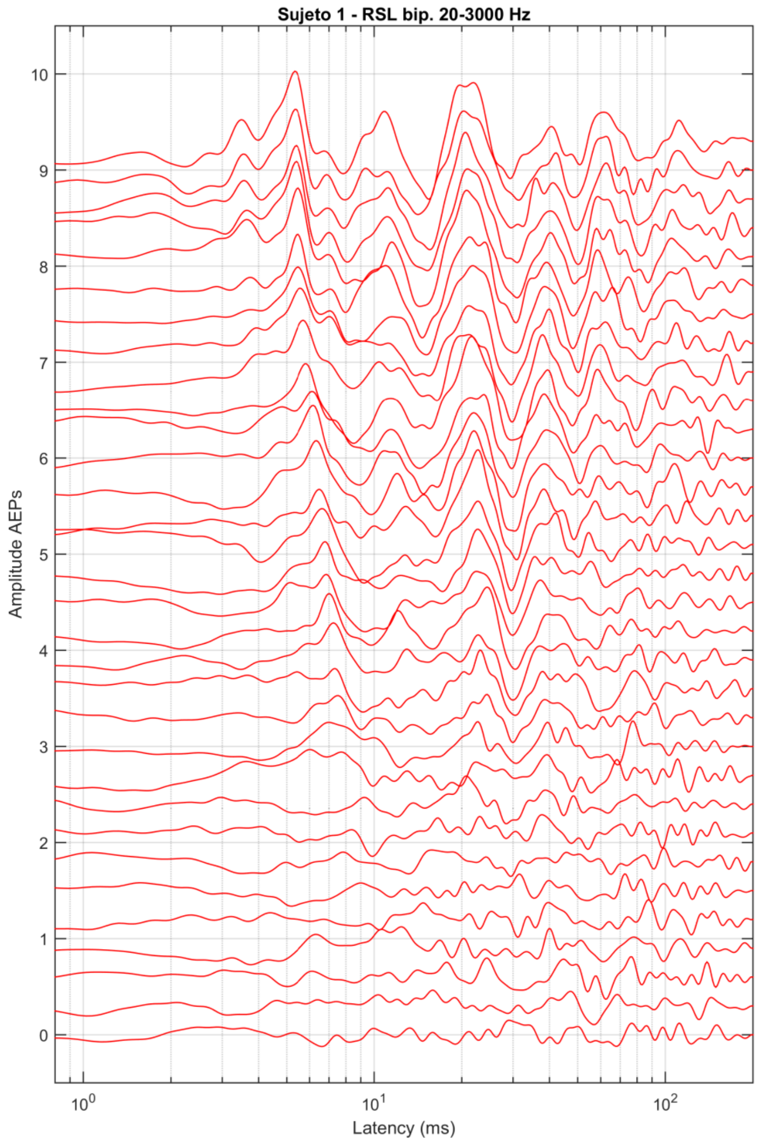


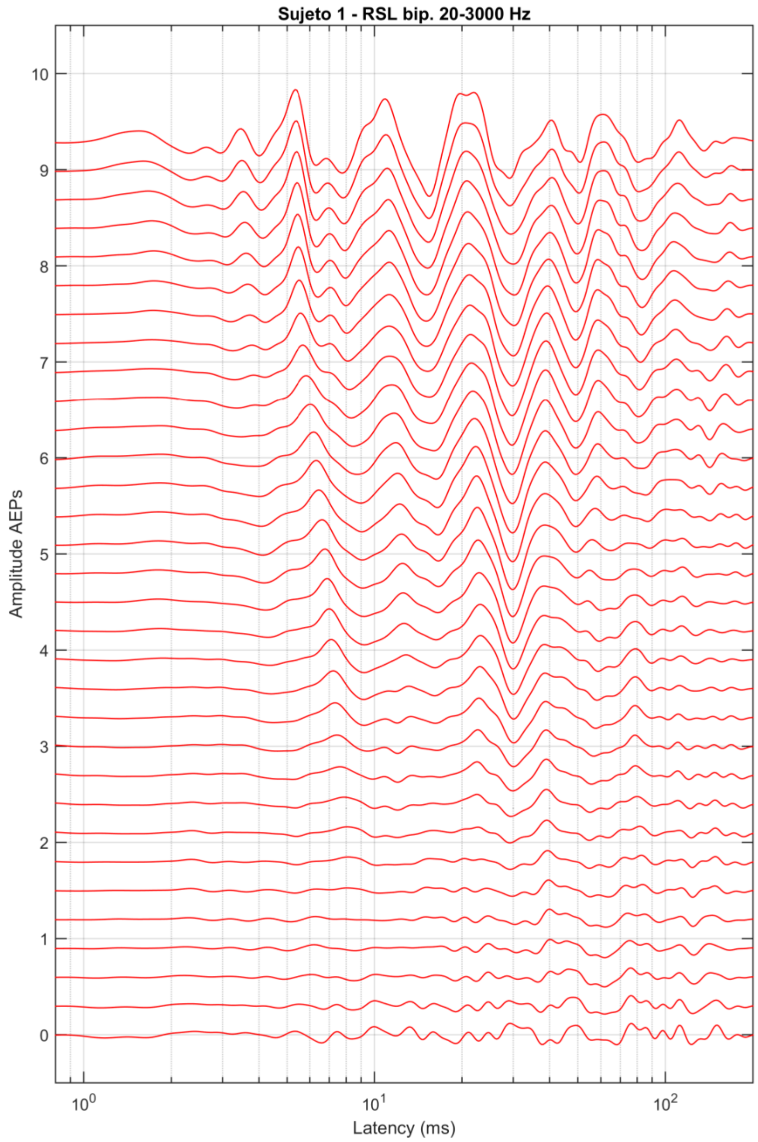
Individual responses: subject 4



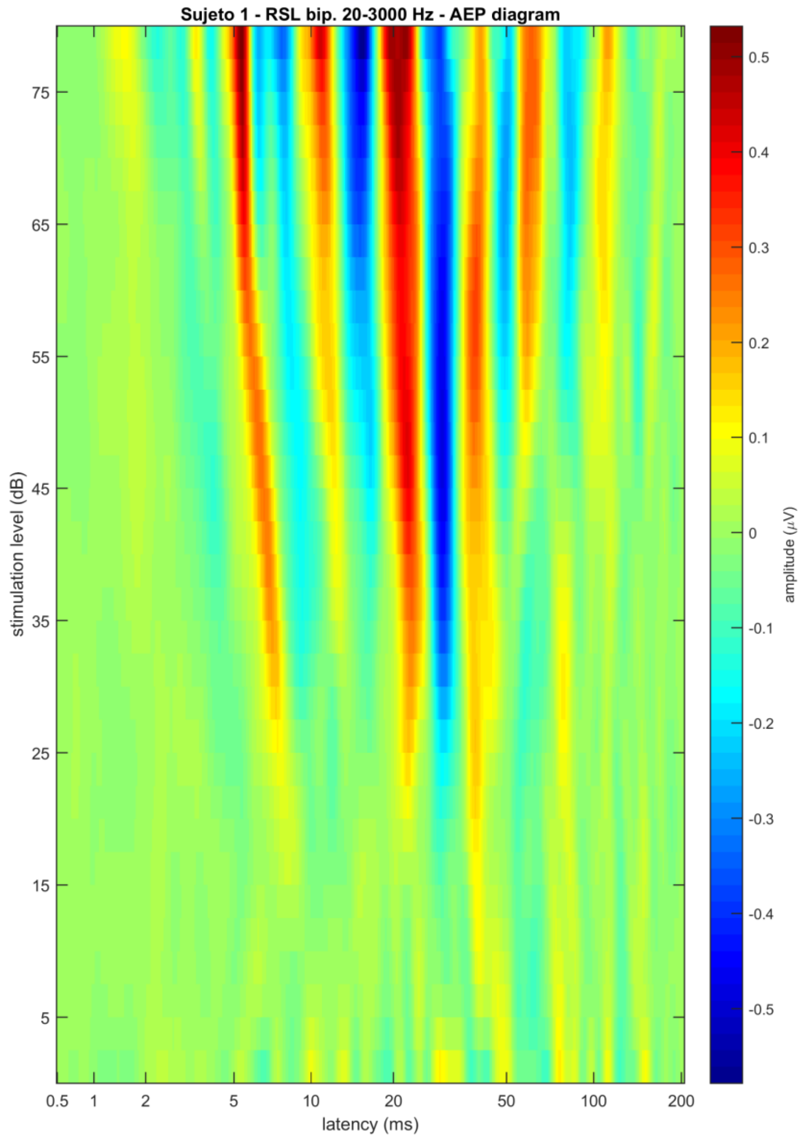
- **Number of categories M can be selected offline**
- **More categories:**
  - **Estimated responses more affected by noise**
  - **Better resolution**
  - **Deconvolution procedure requiring more computer resources, but affordable in reduced representation space**
- **There are solutions for noise reduction and high resolution (future work)**

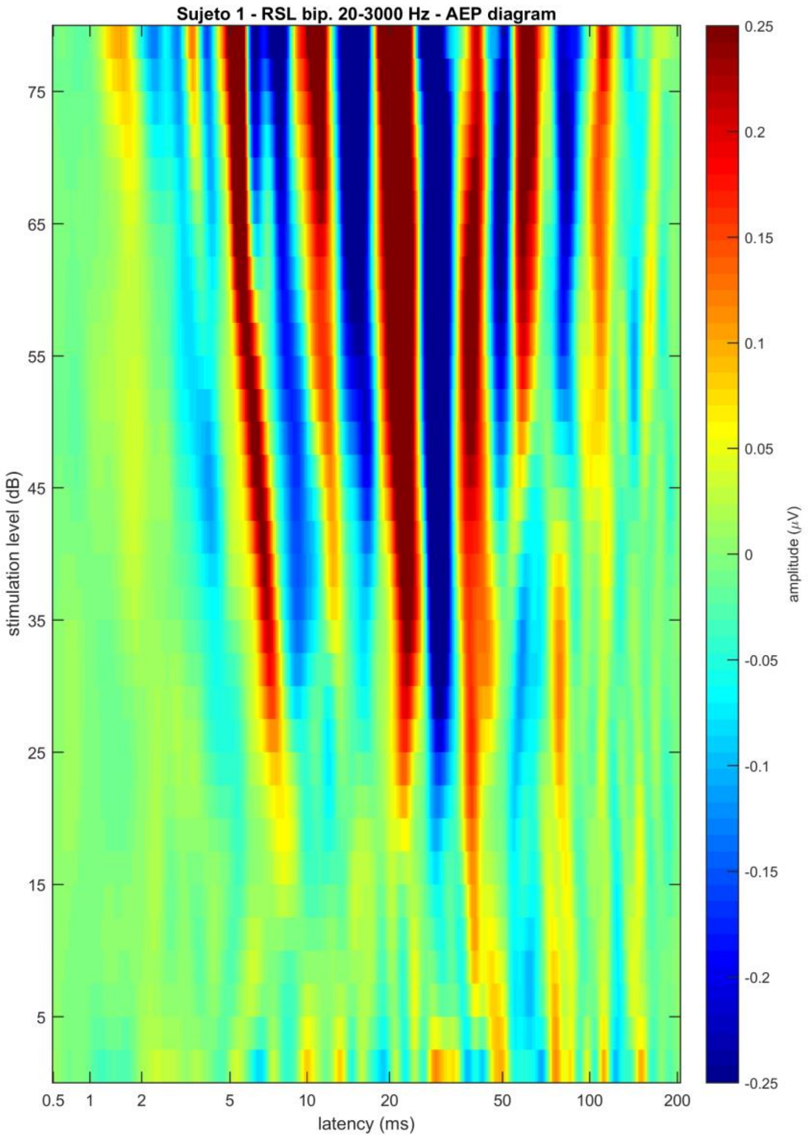
High resolution ABR with multi-response deconvolution





High resolution ABR with multi-response deconvolution





Conclusions

- **Multi-response deconvolution of AEPs is possible in a reduced representation space**
  - **Mathematic fundamental are not very difficult**
  - **Practical problems are identified and controlled**
- **Experiments in this work are relatively simple**
  - **Categorization by stimulation level**
- **Multi-response model allow a broad range of new experimental designs**
  - **Complex sounds, structured stimulation patterns**
- **Clinical and research applications**

Signal Processing in Audiology

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RELEVANT INFORMATION:

- **Multi-response deconvolution provides flexibility in AEP recording**
- **Problem with dimensionality growth (*J x M*)**
- **Not a problem in a reduced representation space (J_r *x M*)**

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In memoriam

Professor José Carlos Segura Luna

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